SUMMER MEETING AT CHICAGO
CONFERENCE REPORTS

CONFERENCE ON ALGEBRA AND TWENTY-THIRD COLLOQUIUM

The Conference on Algebra at the 1941 Summer Meeting of the Society in Chicago was held in three sessions. The first dealt with abstract algebra, especially with lattice theory, and the lectures were given by Professors John von Neumann and Garrett Birkhoff. The session was preceded by the first Colloquium Lecture of Professor Oystein Ore on the allied topic of Mathematical Relations and Structures. The second session was concerned with topics in linear algebra and the theory of matrices and the speakers were Professors Nathan Jacobson and N. H. McCoy. General arithmetic notions came to the fore in the third and final session, with talks by Professors J. F. Ritt and Oscar Zariski, and Dr. O. F. G. Schilling. There was considerable discussion, both organized and spontaneous, after all the talks.

1. *Abstract algebra and lattices*

A number of different algebraic systems can be subsumed under the notion of a lattice-ordered group (or an *l-group*). Professor Birkhoff's talk dealt with the structural properties of such groups. By definition, an l-group is a group which is also a lattice and which has the "homogeneity" property ($x\leqq y$ implies $a+x\leqq a+y$ and $x+a\leqq y+a$). Examples include the additive groups of ordered fields, partially ordered function spaces (Kantorovitch and others), and the lattice of all ideals in an integral domain (Clifford, Lorenzen, Krull).

After discussing the elementary properties of l-groups and the variety of possible postulate systems, Birkhoff turned to their structure theory. The study of homomorphisms of l-groups leads naturally to a concept of an l-ideal. The lattice of all l-ideals is distributive. A central result asserts that every l-group with a chain condition is either directly decomposable, or has exactly one maximal proper l-ideal.

Many of the results known for abelian l-groups hold without serious change for non-abelian groups. However, any l-group in which there is a chain condition for the elements is necessarily abelian. There is a number of unsolved questions: Does there exist a non-abelian l-group which is complete, in the sense that every bounded set of elements has a least upper bound? Is there a theory of l-rings? Are there simple (non-abelian) l-groups which are not simply ordered?

von Neumann began by a comparison of classical and quantum