FORCED OSCILLATIONS OF CONTINUOUS DYNAMICAL SYSTEMS

W. H. INGRAM

Introduction. The system of differential equations and boundary conditions

$$\text{(a)} \qquad \frac{\partial u_i}{\partial x} = \Sigma a_{ij}(x) \frac{\partial u_j}{\partial t} + \Sigma b_{ij}(x) u_j, \quad \Sigma \{ \alpha_{ij} u_j(a) + \beta_{ij} u_j(b) \} = 0$$

$(i, j = 1, 2, \cdots, n, a \leqq x \leqq b)$ has application in the theory of the electrical transmission line, the diffusion of heat along thin rods and around thin rings and, when some of the u's are employed to designate rates of change of other u's, to vibrating strings, bars, air columns and other dynamical systems. The system of total differential equations

$$\text{(b)} \qquad Y'(x) = (\mu \mathcal{A} + \mathcal{B})Y, \qquad \mathcal{W}_a Y(a) + \mathcal{W}_b Y(b) = 0,$$

where $\mathcal{A} = (a_{ij})$, $\mathcal{B} = (b_{ij})$, $\mathcal{W}_a = (\alpha_{ij})$, $\mathcal{W}_b = (\beta_{ij})$, and where Y is a columnar matrix of n elements each a function of x, may be obtained as the result of the Bernoulli-Taylor substitution $u_i(x, t) = \epsilon^{\mu t} y_i(x)$ into (a).

The system (b) has been the starting point for many researches centered around the problem of expressing an arbitrary function f or, more generally, a set of functions $\{f_i\}$, in terms of its characteristic solutions. A solution of this problem in the simple case, having application to the uniform dissipationless vibrating string, was first obtained by Daniel Bernoulli about the year 1732 and a solution having application to the nonuniform string was first obtained by Liouville[1] one hundred years later. A purportedly more rigorous treatment of Liouville's problem was given by Kneser[2] in 1904. Since that date a great many papers have appeared, having to do with the system (b) under one restriction or another, the most comprehensive of which are the papers by Bliss,[3] who obtained uniform convergence in his expansion theorem by requiring (b) to be "definitely" self-adjoint and by imposing a restriction on the functions f_i, and Birkhoff and Langer[4] who considered the general case.

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[1] Liouville, Journal de Mathématiques Pures et Appliquées, vol. 1 (1836), pp. 253, 269.

[2] Kneser, Mathematische Annalen, vol. 58 (1904), p. 108.

[3] Bliss, Transactions of this Society, vol. 28 (1926), p. 576.

[4] Birkhoff and Langer, Proceedings of the American Academy of Arts and Sciences, vol. 58 (1923), p. 100.