

Now, if  $m$  is odd, then  $(m + 1 - 2^{N-3})/2$  is an integer which may be taken as the value of  $b$  since it satisfies the conditions (7). It is easily seen that the set (6), in which  $b = (m + 1 - 2^{N-3})/2$  and  $c$  is determined by (8), is not the set (4). However, if  $m$  is even and not equal to  $2^{N-3}$ , then  $b = (m + 2 - 2^{N-3})/2$  and  $c$  determined by (8) are integers which satisfy (7) and yield a set (6) which is not the set (4). But if  $m = 2^{N-3}$ , then  $N \geq 6$  and  $b = 2$  satisfies the conditions (7) on  $b$  and yields a set (6) which is not the set (4).

An interesting choice of integers  $b$  and  $c$  is that given by  $b = (m + 1 - 2^{N-4})/2$  if  $m$  is odd and less than or equal to  $(2^{N-4} + 2^{N-3} - 1)$ , but by  $b = (m - 2^{N-4})/2$  if  $m$  is even and less than or equal to  $(2^{N-4} + 2^{N-3})$ . Then (6) require no rearrangement, and  $c$  is respectively  $b$  or  $b + 1$ . The resulting integers (6) differ from (4) when  $m \neq 2^{N-4} + 2^{N-3} - 1, 2^{N-4} + 2^{N-3}$ .

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## HAUSDORFF METHODS OF SUMMATION WHICH INCLUDE ALL OF THE CESÀRO METHODS

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### 1. Introduction. The transformation<sup>1</sup>

$$\sigma_m = \sum_{n=0}^m C_{m,n} \Delta^{m-n} c_n \cdot s_n,$$

where  $c_n = \int_0^1 u^n d\phi(u)$  and  $\{s_n\}$  is a given sequence, defines a regular method of summation of the sequence  $\{s_n\}$  provided that  $\phi(u)$  is of bounded variation on the interval  $0 \leq u \leq 1$ , continuous at  $u = 0$ , and

$$\phi(u) = \begin{cases} 0 & \text{if } u = 0, \\ 1 & \text{if } u = 1, \\ \frac{1}{2}[\phi(u - 0) + \phi(u + 0)] & \text{if } 0 \leq u < 1. \end{cases}$$

If these conditions of regularity are fulfilled the sequence  $\{c_n\}$  is said to be a *regular moment sequence* (briefly a *regular sequence*), the mass function  $\phi(u)$  is said to be a *regular mass function*, and the method of summation involved is called a *Hausdorff method of summation* ([1] or [2]) and is designated by the symbol  $[H, \phi(u)]$ .

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<sup>1</sup> To define the symbolism used here we write  $C_{m,n} = m(m-1) \cdots (m-n+1)/n!$ ,  $C_{m,0} = 1$ ;  $\Delta^i x_j = x_j - C_{i,1}x_{j+1} + C_{i,2}x_{j+2} + \cdots$ .