25. L. R. Wilcox: *Extensions of semi-modular lattices.* III.

The author's result (abstract 47-5-208) is extended to all complemented semi-modular lattices of dimension equal to or greater than 4. The following theorem is proved. Let L be left complemented (abstract 47-9-356) with the further property that $b, c \in L$, $bc \neq 0$ implies $(a+b)c = a+bc$ for $a \leqq c$; suppose also that there exists in L a chain of length 6. Then there exists a complemented modular lattice Λ containing L order-isomorphically and having the properties (a) $a \in L$, $a \neq 0$, $b \geqq a$ implies $b \in L$, and (b) for $a \in \Lambda$, $b \in L$, $a \leqq b$ there exists $c \in L$ such that c is a complement (in Λ) of a in b. Properties (a), (b) characterize Λ uniquely up to isomorphisms. This theorem is a lattice-theoretic generalization of well known imbeddings of affine and hyperbolic spaces into projective spaces. (Received November 24, 1941.)

26. Leonard Carlitz: *q-Bernoulli numbers and polynomials.*

Rational functions of q are defined by means of $q(qb+1)^m = b^m$ $(m > 1)$, where after expansion b^m is replaced by b_m; "polynomials" are defined by $b_m(x) = \sum_{\alpha=0}^{m} C_{m,\alpha} q^{\alpha x} [x]^{m-\alpha} b_\alpha$, where $[x] = (q^x - 1)/(q-1)$. Many of the properties of the ordinary Bernoulli numbers and polynomials are readily extended to these quantities; in addition there are certain formulas in the generalized case that are not easily specialized to the case $q=1$. Among possible explicit formulas for b_m may be mentioned $b_m = \sum_{s=0}^{m} 1/[s+1] \sum_{\alpha=0}^{s} (-1)^\alpha \left[{s \atop \alpha}\right] q^{\alpha(\alpha+1-2s)/2}$. $[\alpha]^m$, which leads at once to a generalized Staudt-Clausen theorem: $b_m = \sum_{s=2}^{m+1} N_s(q)/F_s(q)$ $(m > 0)$, where $F_s(q)$ is the cyclotomic polynomial and deg $N_s <$ deg F_s. (Received November 24, 1941.)

27. Joseph Lehner: *The Ramanujan identities and congruences for powers of eleven.* Preliminary report.

The author proves the existence of a "Ramanujan identity" for the modulus 11^α $(\alpha \geqq 1)$. For $\alpha = 1, 2$, this identity implies the Ramanujan conjecture: $p(n) \equiv 0$ (mod 11^α) if $24n \equiv 1$ (mod 11^α). The methods used are those of Rademacher's paper *The Ramanujan identities under modular substitution* (to be published in the American Journal of Mathematics). A modification of Hecke's T-operator is used. This operator is defined as follows: $U_{11}F(\tau) = \Sigma_\lambda F(\tau + \lambda/11)$, λ mod 11. If $F(\tau)$ is a modular function belonging to $\Gamma_0(121)$, that subgroup of the modular group defined by $c \equiv 0$ (mod 121), then $U_{11}F$ belongs to $\Gamma_0(11)$. Then it can be expressed as a polynomial in $A(\tau)$, $B(\tau)$, certain well known functions which constitute a basis for $\Gamma_0(11)$. By taking F to be $\eta(121\tau)/\eta(\tau)$, where $\eta(\tau)$ is the well known elliptic modular function of Dedekind, we obtain the desired Ramanujan identity for the modulus 11. Identities for higher powers of 11 are then obtained by a two-fold induction, one for even α, the other for odd α. The possibility of proving Ramanujan's conjecture for higher values of α $(\alpha > 2)$ is being investigated. (Received November 19, 1941.)

Analysis

28. C. B. Barker: *The Lagrange multiplier rule for two dependent and two independent variables.*

Let $\bar{z}_1(x, y)$ and $\bar{z}_2(x, y)$ be of class C'''' on a closed simply connected region $\overline{G}$ of class C_α''' and minimize (1) $\iint_G f(x, y, z_1, z_2, p_1, p_2, q_1, q_2)dxdy$ among all pairs of functions $z_1(x, y)$ and $z_2(x, y)$ which coincide on the boundary G^* with $\bar{z}_1(x, y)$ and $\bar{z}_2(x, y)$, respectively, and which satisfy (2) $\phi(x, y, z_1, z_2, p_1, p_2, q_1, q_2) = 0$ on G; assume that f