HEAT CONDUCTION IN AN INFINITE COMPOSITE SOLID[1]

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1. **Introduction.** The problem of one-dimensional heat conduction in finite or semi-infinite composite solids has been investigated extensively. The doubly-infinite case, however, seems to have been treated only for special initial temperature distribution functions.[2]

The purpose of the present paper is to treat the general case. The Laplace transformation is used formally in §2 to discover the solution, which is then rigorously established in §3. In §4, finally, a uniqueness theorem is proved, under more restrictive conditions on the initial distribution function.

Lebesgue integrals are used throughout.

2. **The formal solution.** Consider two plane-boundary semi-infinite homogeneous solids, composed of different materials, placed in perfect thermal contact. If the conduction of heat takes place in only one dimension, perpendicular to the interface, the temperature $U(x, t)$ satisfies the following differential system:

$$\frac{\partial U}{\partial t} = a_\nu \frac{\partial^2 U}{\partial x^2}; \qquad t > 0;\ x < 0,\ \nu = 1;\ x > 0,\ \nu = 2; \tag{1}$$

$$\lim_{t\to 0} U(x, t) = f(x); \qquad x \neq 0; \tag{2}$$

$$\lim_{x\to -0} U(x, t) = \lim_{x\to +0} U(x, t); \qquad t > 0; \tag{3}$$

$$\lim_{x\to -0} k_1 \frac{\partial U}{\partial x} = \lim_{x\to +0} k_2 \frac{\partial U}{\partial x}; \qquad t > 0; \tag{4}$$

where x is the perpendicular distance from the interface, t is time, a_ν and k_ν are the thermal diffusivities and conductivities, respectively, of the two materials and are positive constants, and $f(x)$ is a known function, defined for all real x except $x=0$, whose properties will be specified later.

Denoting the common limit in equation (3) by $\Phi(t)$, the solution can be written in the well known form[3]

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[1] Presented to the Society, April 5, 1941.

[2] Cf. Riemann-Weber, *Die partiellen Differential-Gleichungen der mathematischen Physik*, 5th edition, 1912, vol. 2, p. 98.

[3] Cf. H. S. Carslaw, *The Mathematical Theory of the Conduction of Heat in Solids*, 2d edition, London, 1921, §§18, 23.