SOME APPLICATIONS OF CERTAIN POLYNOMIAL CLASSES[1]

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The term *applications* will be construed broadly enough as to include *properties*. It is here proposed, then, to examine some properties and some applications of some classes of polynomial sets.

1. **Formal properties of Appell sets.** The power series approach to the theory of analytic functions presents us with a very simple polynomial set.[2] Thus, if $f(x)$ is analytic about $x=a$, we have the expansion

$$(1) \qquad f(x) = \sum_{n=0}^{\infty} c_n \frac{(x-a)^n}{n!}, \qquad c_n = f^{(n)}(a),$$

in terms of the set

$$\{(x-a)^n/n!\}.$$

This set has the important property of reproducing itself under the operation of differentiation, in accordance with the rule.

$$(2) \qquad \frac{d}{dx} P_n(x) = P_{n-1}(x),$$

where

$$P_n = (x-a)^n/n!.$$

Now this set is not uniquely determined by (2). There are in fact infinitely many sets of polynomials $\{P_n\}$ that satisfy (2). These bear the name of *Appell sets*,[3] after the man who in 1880 [1][4] first made a study of them. Appell sets will be the first class to be considered here.

There are many conditions that are equivalent to the defining relation (2) for Appell sets. Among the simplest are the following two:

[1] An address delivered before the meeting of the Society in Washington, D.C., May 3, 1941, by invitation of the Program Committee.

[2] By a set $\{P_n\}$ we understand an infinite sequence $P_0, P_1, \cdots$, with P_n of degree n.

[3] Strictly speaking, Appell's definition is

$$(2') \qquad \frac{d}{dx} P_n(x) = nP_{n-1}(x),$$

but (2) is preferable for our purpose. If $\{P_n\}$ satisfies (2), then $\{n!\,P_n\}$ satisfies (2′), and conversely. A bibliography on Appell polynomials is given in Davis [5].

[4] Numbers in brackets refer to the bibliography placed at the end.