Geometry

319. J. J. DeCicco: *Equilong geometry of series of collineal third order differential elements.*

The direct equilong group induces at a fixed line (u, v) of the plane a six-parameter group G_6 between the differential elements of third order. Any three elements (C_1, C_2, C_3) possess the fundamental invariant $(\delta_1 r_1+\delta_2 r_2+\delta_3 r_3)^2/(\delta_1 s_1+\delta_2 s_2+\delta_3 s_3)$, where δ_k is the distance from the point of C_i to that of C_j, r_i is the radius of curvature of C_i and $s_i=dr_i/d\theta_i$ is the rate of variation of the radius of curvature per unit radian measure of the inclination θ_i at the fixed line. In general, n elements possess $3n-6$ independent invariants, of which $n-1$ are distances, and the remaining $2n-5$ are of the new type given above. Any other invariant of these n elements must be a function of these $3n-6$ independent invariants. A series $r=r(w)$, $s=s(w)$ possesses the two fundamental differential invariants $k_1=(d^2r/dw^2)/(d^3r/dw^3)$, and $k_2=(d^2r/dw^2)^2 \div (d^2s/dw^2)$. Any two series with their curvatures the same functions of the distance w are equivalent under our group G_6. (Received May 21, 1941.)

320. Edward Kasner and J. J. DeCicco: *Conformal geometry of series of third order differential elements.*

Kasner and Comenetz have shown that at a fixed point of the plane the direct conformal group induces a six-parameter group G_6 between the differential elements of third order. In this paper it is found that three elements (C_1, C_2, C_3) possess the fundamental invariant $(k_1 \sin a_1+k_2 \sin a_2+k_3 \sin a_3)^2/(l_1 \sin 2a_1+l_2 \sin 2a_2+l_3 \sin 2a_3)$, where a_λ is the angle from C_μ to C_ν, k_λ is the curvature of C_λ, and $l_\lambda=dk_\lambda/ds_\lambda$ is the rate of variation of the curvature per unit length of arc at the fixed point. In general, n elements possess $3n-6$ independent invariants, of which $n-1$ are angles and the remaining $2n-5$ are of the new type given above. Any other invariant of these n elements must be a function of these $3n-6$ independent invariants. A series $k=k(\theta)$, $l=l(\theta)$ possesses the two fundamental differential invariants $\rho_1=(d^2k/d\theta^2+k) \div (d^3k/d\theta^3+dk/d\theta)$ and $\rho_2=(d^2k/d\theta^2+k)^2/(d^2l/d\theta^2+4l)$. Any two series with the curvatures ρ_1 and ρ_2 the same functions of the inclination θ are equivalent under the Kasner-Comenetz group G_6. (Received May 21, 1941.)

321. Edward Kasner and J. J. DeCicco: *General differential geometry of second order differential elements.*

Kasner (American Journal of Mathematics, vol. 28, pp. 203–213) showed that at a fixed point of the plane the entire group of arbitrary point transformations induces an eight-parameter group G_8 between differential elements of second order. He gave a complete discussion of all the invariants of n elements with all the appropriate geometric interpretations. This new paper considers the differential geometry of a series $q=q(p)$ under this group G_8, where $p=dy/dx$ and $q=d^2y/dx^2$. The length of arc of a series is $s=\int[5q^{\text{iv}}q^{\text{vi}}-6(q^{\text{v}})^2]^{1/2}/q^{\text{iv}}dp$, and the curvature is $K=[25(q^{\text{iv}})^2q^{\text{vii}}-105q^{\text{iv}}q^{\text{v}}q^{\text{vi}}+84(q^{\text{v}})^3]/[5q^{\text{iv}}q^{\text{vi}}-6(q^{\text{v}})^2]^{3/2}$, where the superscripts denote differentiation with respect to p. Any two series with their curvatures K the same functions of the arc length s are equivalent under the group G_8. (Received May 21, 1941.)

322. D. T. McClay: *Clifford numbers.*

The system of numbers $a+b\Lambda$, with a and b real and $\Lambda^2=1$, first used by Clifford, is an algebra of Weierstrass. If these "Clifford numbers" are represented by points