

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

ALGEBRA AND THEORY OF NUMBERS

293. A. H. Clifford: *Matrix representations of completely simple semigroups.*

By a representation \mathfrak{T}^* of degree n of a semigroup S is meant a mapping $a \rightarrow T^*(a)$ of S into the set of all square matrices of n rows and columns such that $T^*(ab) = T^*(a)T^*(b)$. A. Suschkewitsch (Communications de la Société Mathématique de Kharkow, vol. 6 (1933), pp. 27–38) partially solved the problem of determining all representations of a type of semigroup which he calls a “Kerngruppe.” In the terminology of D. Rees (Proceedings of the Cambridge Philosophical Society, vol. 36 (1940), pp. 387–400), the latter is a “completely simple semigroup without zero.” The present paper completes the solution, at the same time allowing S to be any completely simple semigroup. To any such S belongs a “structure group” G , and every representation \mathfrak{T}^* of S is an “extension” of a representation \mathfrak{T} of G . To any given \mathfrak{T} corresponds a certain matrix H , possibly infinite. \mathfrak{T} admits an extension \mathfrak{T}^* of finite degree if and only if H has finite rank h . If \mathfrak{T} has degree n , there exists a “basic” extension \mathfrak{T}_0^* of degree $n+h$, and every extension \mathfrak{T}^* of \mathfrak{T} , even though it may be indecomposable, reduces into \mathfrak{T}_0^* and null representations. (Received March 31, 1941.)

294. C. J. Everett: *Vector spaces over rings.*

Every basis of a vector space V_n (with n basis elements) over a ring K with right ideal maximal condition has n elements. A ring is exhibited over which any V_n is always a V_1 . Every submodule M of a V_n over ring K has a basis of $b(M) \leq n$ elements if and only if (property P) every right ideal of K is principal of type rK , where r is a non-left-zero-divisor. For V_n over K with property P , $b(M)$ is a positive modular functional (G. Birkhoff, *Lattice Theory*, American Mathematical Society Colloquium Publications, vol. 25, p. 40). If K has property P and is without zero-divisors, the metric homomorph (loc. cit., Theorem 3.10) of the lattice of submodules of V_n over K is lattice-isomorphic with the lattice of submodules of V_n over the quotient field of K . For V_n over K of type P , $b(M)$ is sharply positive (loc. cit., p. 41) if and only if K is a quasi-field. (Received May 26, 1941.)

295. A. W. Jones: *On the characterization of groups whose lattices satisfy specified lattice identities.*

It is shown that the lattice of a group is simple, complemented, and modular (that is, capable of abstractly representing a finite projective geometry) if and only if the group is either: (1) abelian of prime power order and with prime order elements