A NOTE ON QUASI-METRIC SPACES[1]

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1. **Introduction.** In a recent abstract[2] the author introduced an important application of a class of neighborhood spaces in which pairs of points are required to satisfy only a very weak separation axiom (due to Kolmogoroff):

(K) *If x and y are distinct points, then at least one of these points has a neighborhood which does not contain the other.*

In the present note it is shown that a very useful generalized distance function may be defined in certain of these spaces.

Clearly, any such distance function must be an asymmetric one. W. A. Wilson[3] considered the definition of asymmetric distances in certain spaces which satisfy stronger separation axioms than K. It is shown here that a slight modification of one of the axioms in [W] allows the extension of a large part of the theory developed there to spaces subject to K.

Since many of the theorems and proofs in [W] remain valid here with only very obvious changes, this note will be limited to a mere sketch concerning new properties which arise under the weaker axioms used. The reader should have no difficulty in adapting the more complete discussion given in [W] to the case studied here.

2. **The distance function.** The symbol 1 will denote a space of points; points and point sets in 1 will be denoted by small and capital letters respectively.

The space 1 will be said to be *quasi-metric* if for every pair of points x, y in 1 there are defined two non-negative numbers xy and yx, not necessarily equal, which satisfy the following postulates:

I. $xy=yx=0$ *if and only if* $x=y$;[4]
II. *for any three points*, $xy \leqq xz+zy$.

If for some pair of points $xy=0\neq yx$, then the point x will be said to be *adjacent* to the point y. It is to be emphasized that, regardless

[1] Presented to the Society, April 13, 1940.

[2] Abstract 46-3-138, this Bulletin.

[3] W. A. Wilson, *On quasi-metric spaces*, American Journal of Mathematics, vol. 53 (1931), p. 675. Hereafter this paper will be referred to as [W].

[4] In [W] the stronger axiom: $xy=0$ if and only if $x=y$ was used. This excludes the case $xy=0\neq yx$ allowed here.