SOME RECENT DEVELOPMENTS IN THE THEORY OF CONTINUED FRACTIONS[1]

H. S. WALL

The principal rôle of the continued fraction in analysis has perhaps been that of an intermediary between more familiar and easily handled things, such as between the power series and the integral. This may partly explain the fact that there are certain questions about continued fractions which have remained relatively unexplored. To illustrate what I mean, if one's principal attention were focused upon power series, and continued fractions were used only incidentally, it is unlikely that one would imagine that the convergence region for the continued fraction is, as it now appears, more properly a *parabolic* region than a circular region.

I wish to speak today about some results which have been obtained during the last few years, by a group of men with whom I have been associated. Our investigations have been centered mainly upon the continued fraction itself. In certain instances it has been possible to apply our results to problems not directly connected with continued fractions. Thus, during the course of this lecture I shall have occasion to speak of the problem of moments, of Hausdorff summability, and of certain classes of analytic functions.

1. Some definitions and formulas.

Before discussing some of the problems with which we have been concerned, I shall put down some necessary definitions and formulas. The continued fractions considered are chiefly of the form

$$\frac{1}{1} + \frac{a_2}{1} + \frac{a_3}{1} + \frac{a_4}{1} \quad \cdots \tag{1.1}$$

in which $a_2, a_3, a_4, \cdots$ are complex numbers. Apart from unessential initial irregularities, any continued fraction in which the partial denominators are different from zero can be thrown into this form. The nth *approximant* of (1.1) is the ordinary fraction, A_n/B_n, obtained by stopping with the nth partial quotient. The *numerators* and *denominators* may be computed by means of the recursion formulas

$$\begin{aligned} &A_0 = 0, \quad B_0 = 1, \quad A_1 = 1, \quad B_1 = 1, \\ &A_n = A_{n-1} + a_n A_{n-2}, \quad B_n = B_{n-1} + a_n B_{n-2}, \quad n = 2, 3, 4, \cdots . \end{aligned} \tag{1.2}$$

[1] An address delivered before the Detroit meeting of the Society, November 23, 1940, by invitation of the Program Committee.