$$(A + B)C \leqq AC + BC,$$

and our theorem is proved.

We can also prove the following:

3.2. COROLLARY. *A necessary and sufficient condition that*

$$(A + B)C = AC + BC$$

for positive A, B, and C is that either $C=1$, or $1<C<\omega$ and $\alpha_0 \leqq \beta_0$, or $\omega \leqq C$ and $\alpha_0+\gamma_0<\beta_0+\gamma_0$.

This corollary follows quite easily from the reasoning found in the preceding section.

CORNELL UNIVERSITY

THE DECOMPOSITION THEOREM FOR ABELIAN GROUPS[1]

JOEL BRENNER

Let G be an abelian group such that $p^k g=0$ for all $g\in G$, p prime, k fixed. We prove G *has a basis*, that is, a set of elements such that each $g\in G$ is uniquely expressible as a linear combination of elements of the set.[2]

THEOREM. *There exists an ascending chain of sets B_i, $0\leqq i\leqq k$, of elements of G with the properties*:

(i) *Every element in B_i is of order greater than p^{k-i}.*

(ii) *The elements in B_i are completely linearly independent.*

(iii) *If the order of the element g in G is greater than p^{k-i}, then there exists a (unique) linear combination z of elements of B_i such that the order of $g-z$ is at most p^{k-i}.*

Since we may choose as B_0 the vacuous set, we may assume that the sets $B_0, \cdots, B_s$ have already been constructed in such a way as to meet the requirements (i) to (iii). In order to construct B_{s+1} we adjoin to B_s any greatest subset C of G with the following properties.

(a) All the elements in C are of order p^{k-s}.

(b) The join B_{s+1} of the sets B_s and C is an independent set.

---

[1] Presented to the Society, April 6, 1940.

[2] Unique in that the number of nonzero terms in an expression for g is unique and only the arrangement but not the respective values of the nonzero terms may differ in two expressions for g.