# MAXIMUM OF CERTAIN FUNDAMENTAL LAGRANGE INTERPOLATION POLYNOMIALS[1]

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This note extends some of the results obtained in a previous paper[2] which we shall designate as I. The notations are the same.

We are concerned with the polynomials

$$l_k^{(n)}(x) \equiv \frac{\phi_n(x)}{\phi_n'(x_k)(x - x_k)}, \qquad k = 1, 2, \cdots, n,$$

where $\phi_n(x) \equiv (x-x_1)(x-x_2)\cdots(x-x_n)$ is the Jacobi polynomial of degree $n$ which satisfies the differential equation $(1-x^2)\phi_n''(x) + [\alpha-\beta-(\alpha+\beta)x]\phi_n'(x)+n(n+\alpha+\beta-1)\phi_n(x)=0$. The parameters $\alpha$, $\beta$ are positive and $n$ is a positive integer. It is known that $-1<x_n<x_{n-1}<\cdots<x_1<1$. Throughout the paper, $x$ is always restricted to the interval $-1\leqq x\leqq 1$.

It was shown in I, for example, that, if $\alpha=\beta=\frac{3}{2}$, $\max |l_k^{(n)}(x)| <2$ and $l_1^{(n)}(1)\to 2$ as $n\to\infty$.

Now we use[3]

$$\phi_n(1) = \frac{2^n\Gamma(n+\beta)\Gamma(n+\alpha+\beta-1)}{\Gamma(\beta)\Gamma(2n+\alpha+\beta-1)}$$

and the asymptotic expressions[4]

$$\phi_n(\cos\theta) = \frac{2^n\Gamma(n+1)\Gamma(n+\alpha+\beta-1)}{(\pi n)^{1/2}\Gamma(2n+\alpha+\beta-1)}\left(\sin\frac{\theta}{2}\right)^{1/2-\beta}\left(\cos\frac{\theta}{2}\right)^{1/2-\alpha}$$
$$\cdot\left\{\cos\left[N\theta-(2\beta-1)\pi/4\right]+(n\sin\theta)^{-1}O(1)\right\},$$

$$\phi_n(\cos\theta) = \frac{2^n\Gamma(n+1)\Gamma(n+\alpha+\beta-1)}{\Gamma(2n+\alpha+\beta-1)}\left(\sin\frac{\theta}{2}\right)^{1-\beta}\left(\cos\frac{\theta}{2}\right)^{1-\alpha}$$
$$\cdot\left\{\frac{\Gamma(n+\beta)}{\Gamma(n+1)}\left(\frac{\theta}{\sin\theta}\right)^{1/2}\frac{J_{\beta-1}(N\theta)}{N^{\beta-1}}+\theta^{1/2}O(n^{-3/2})\right\},$$

where $N=n+(\alpha+\beta-1)/2$, $cn^{-1}\leqq\theta\leqq\pi-\epsilon$, $c$, $\epsilon$ positive constants and

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[1] Presented to the Society, April 13, 1940.

[2] M. Webster, *Note on certain Lagrange interpolation polynomials*, this Bulletin, vol. 45 (1939), pp. 870–873.

[3] C. Winston, *On mechanical quadratures formulae involving the classical orthogonal polynomials*, Annals of Mathematics, (2), vol. 35 (1934), pp. 658–677.

[4] G. Szegö, *Orthogonal Polynomials*, American Mathematical Society Colloquium Publications, vol. 23, 1939, pp. 191–192, 121, 123.