BOOK REVIEWS

La Théorie des Spineurs. By Élie Cartan. (Actualités Scientifiques et Industrielles, nos. 643 and 701.) Paris, Hermann, 1938. 95 pp. and 91 pp.

The theory of spinors has been treated in the literature from three points of view: the infinitesimal, the algebraic and the geometric. Cartan originated the theory in its infinitesimal aspect in 1913 and he has now written a full length account in which it has been his intention "de développer systematiquement la théorie des spineurs en donnant de ces êtres mathématiques une définition purement géométrique."

Cartan's definition of a spinor (§§93 and 109) is based upon the one-to-two correspondence between the ν-vectors which are coordinates of linear spaces of ν dimensions on the fundamental quadric cone in 2ν (or $2\nu+1$) dimensions and certain vectors with 2^ν components, called "simple" spinors. Despite this geometric introduction to the concept of a spinor, knowledge of the algebraic theory of the representation of linear groups will be of more use to the reader than any acquaintance with the classical ideas of geometry.

The first two chapters of the first volume do not contain the word "spinor." Instead, they include an elementary discussion of the following topics: euclidean space (including the complex and the real definite and indefinite cases), rotations and reflections, multivectors, definition of tensors, tensor algebra, irreducible and reducible tensors, matrices (their algebra and a brief account of unitary, orthogonal and hermitian matrices), and the irreducibility of multivectors. Any reader already acquainted with the material in these forty-seven pages will delight in the concise elegance of the exposition while others will benefit from its lucidity and accuracy.

In the third and fourth chapters the spinor theory is developed in detail for a three dimensional space. The components of a spinor are first introduced as specific functions of the components of an isotropic vector, affected however by an ambiguous sign. After showing that a spinor is a "euclidean tensor," the author introduces in a natural fashion the relation $(x_1)^2+(x_2)^2+(x_3)^2=XX$, where X is a matrix with linear forms in x_1, x_2 and x_3 as elements and the left member is a scalar matrix. This association of vectors and matrices is used to obtain the double-valued representation of the rotation group by linear transformations on spinors. Chapter III concludes with a discussion of reality conditions in the euclidean and pseudo-euclidean