A NOTE ON THE DEFINITION OF ARC-SETS[1]

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The interesting subsets of a Peano space which were later called the arc-sets or A-sets were defined independently and in a different manner by G. T. Whyburn[2] and the author,[3] and each studied the properties of these sets which have proved so useful in the later theory. The similarity of the properties led shortly to the conjecture that a relation existed between the two definitions, and investigation proved that they were equivalent. These are the properties (1) and (3) of the theorem of the present note. Two more equivalent properties, the properties (2) and (4) of the present note, were observed immediately.[4]

The present note introduces two new properties and shows that all six are equivalent, so that any one may be taken as the definition of the arc-sets or A-sets.

THEOREM. *For a nondegenerate*[5] *subset A of a Peano space P the following six properties are equivalent*:

(1) *There is a set H such that A is the set of points of all arcs of P whose end points belong to H.*

(2) *A contains every arc of P whose end points belong to A.*

(3) *A is a connected collection of cyclic elements of P.*

(4) *For any connected subset C of P, the set $A \cdot C$ is connected.*

(5) *Every set separating*[6] *two points of A in A separates them in P.*

(6) *A is connected, $\overline{A} \cdot \overline{C}$ is a single point for each component C of $P-A$, and this point y belongs to A if $\overline{A}$ contains two continua S and T such that $S \cdot T = y$.*

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[1] Presented to the Society, December 1, 1939.

[2] *Concerning the structure of a continuous curve*, American Journal of Mathematics, vol. 50 (1928), pp. 167–194.

[3] *Concerning the arc-curves and basic sets of a continuous curve*, Transactions of this Society, vol. 30 (1928), pp. 567–578.

[4] Properties (1), (3), and (4) were proved equivalent by W. L. Ayres, *Concerning the arc-curves and basic sets of a continuous curve, second paper*, Transactions of this Society, vol. 31 (1929), pp. 595–612. That (1), (2), (3), and (4) are equivalent was remarked with an indication of proof by C. Kuratowski and G. T. Whyburn, *Sur les éléments cycliques et leurs applications*, Fundamenta Mathematicae, vol. 16 (1930), pp. 305–331. See page 321.

[5] Even in the trivial degenerate case, (1), (2), (4), (5), and (6) are still equivalent if we define a point x as being an arc from x to x.

[6] We use the word *separating* here in the weak or *coupure* sense, that is, a set K *separates* two points x and y in a set A if $x+y \subset A$ and there exists no connected set C such that $x+y \subset C \subset A-K$.