ALMOST CYCLIC ELEMENTS AND SIMPLE LINKS OF A CONTINUOUS CURVE[1]

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Many of the definitions and results concerning connected im kleinen continua become useful only when these continua are locally compact. This is especially true in the cyclic element theory. For if a continuous curve M is not locally compact, it is not necessarily true that a simple closed curve in M belongs to a cyclic element of M. Furthermore, if a continuous curve M is not locally compact, it is not necessarily true that a simple closed curve in M belongs to a simple link in M. However, if M is a continuous curve, there are in M certain subcontinua which strongly resemble both cyclic elements and simple links such that if J is a simple closed curve in M, one of them contains J. It is the purpose of this paper to define these sets, to develop a few of their properties, and to show how they are of considerable interest in spaces where the Jordan curve theorem holds true.

1. **Results for complete Moore spaces.** In this section it is assumed that S, the set of all points, is a complete Moore space, that is, Axioms 0 and 1 of R. L. Moore's *Foundations of Point Set Theory*[2] hold true in S.

DEFINITION. *Suppose that K is a nondegenerate subset of a continuous curve M such that* (1) *if A and B are distinct points of K, there exists a simple closed curve lying in M and containing $A+B$, and* (2) *if X is a point of $M-K$, there is some point O of K such that no simple closed curve lying in M contains both X and O. The set K is said to be a "cyclic nucleus"*[3] *of M.*

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[1] Presented to the Society, December 28, 1939.

[2] American Mathematical Society Colloquium Publications, vol. 13, New York, 1932. Hereinafter, this book will be referred to as *Foundations*. The reader is referred to *Foundations* for the definition of terms used, but not specifically defined, here.

[3] A continuous curve is defined to be a connected im kleinen continuum. It need *not* be locally compact. It is easy to see with the help of Theorems 118 and 120 in Chapter I and the arguments for Theorems 6 and 7 in Chapter II of *Foundations* that if a nondegenerate continuous curve M is regarded as a space and the term "region" is interpreted to mean a connected open subset of M, then with respect to this interpretation of "point" and "region," Axioms 0, 1, and 2 of *Foundations* hold true in M and "limit point" is invariant under this change. Hence, it is possible to apply certain theorems found in Chapter II of *Foundations* and elsewhere to continuous curves. For example, any two points of a connected open subset D of a continuous curve are the extremities of an arc lying wholly in D.