

## ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

394. Leon Alaoglu, Garrett Birkhoff, and B. J. Pettis: *General ergodic theorems*. II. Preliminary report.

As a further application of the methods of the paper *General ergodic theorems*, *Annals of Mathematics*, (2), vol. 41 (1940), pp. 293-309, the following theorem is proved. The space  $E$  is a Banach space which is also a complete linear lattice; furthermore  $\| |x| \| = \|x\|$ , and the norm is a monotone non-decreasing function of positive elements. The abelian semi-group  $G$  consists of positive linear transformations on  $E$  to  $E$ ; means of transformations of  $G$  are denoted by  $\tau$ . Then, if  $\limsup_r |\tau x| < +\infty$  for all  $x$  of  $E$ , the limit  $\lim_r \tau x$  exists in order for all  $x$  for which it exists in norm, and the two are equal. These limits of means are defined on page 296 of the above paper. If  $E$  is reflexive, the hypothesis of lattice completeness can be dropped, and the limit in norm exists for all  $x$ . (Received June 19, 1940.)

395. M. M. Day: *Ordered sets and closures*.

A closure function on  $X$  is any function  $c$  defined on all subsets of  $X$  whose values are subsets of  $X$ ;  $c$  is monotone if  $Y \subset Y' \subset X$  implies  $cY \subset cY'$ . A set  $A$  is called effectively ordered if the order relation is transitive and if every element of  $A$  has a successor. A natural definition of convergence in a neighborhood space is given which reduces to the definition of the Moore-Smith limit if  $A$  is directed. Definitions of closure in terms of neighborhoods and of convergence of functions on effectively ordered sets are given and it is shown that the closures defined by either method are the monotone closures. Tukey's definition of order among directed sets is extended to effectively ordered sets and it is shown that much but not all of the theory that holds for directed sets carries over to effectively ordered sets. (Received June 8, 1940.)

396. M. M. Day: *Reflexive Banach spaces which cannot be made uniformly convex*.

The space  $B = (B_1 \times B_2 \times \dots)_p$  (S. Banach, *Théorie des Opérations Linéaires*, Warsaw, 1932, p. 243) is shown to be reflexive if all  $B_i$  are. If  $B_i$  is taken to be the  $i$ -dimensional space with  $\|b\|_{B_i} = \sup |b_j|$  or  $= \sum |b_j|$ ,  $j \leq i$ , where  $b = (b_1, \dots, b_i)$ , it is shown that  $B$  is not isomorphic to any uniformly convex space. By choosing subspaces isomorphic to these it can be shown, for example, that if  $B_i = L^{q_i}$  or  $l^{q_i}$ ,  $1 < q_i < \infty$ , and if there do not exist  $m$  and  $M$  with  $1 < m \leq q_i \leq M < \infty$  for all  $i$ , then  $B$  is not isomorphic to a uniformly convex space. (Received June 10, 1940.)