The Classical Groups. By Hermann Weyl. Princeton, University Press, 1939. 11+302 pp.

It is a curious fact that while almost all the textbooks on higher algebra written prior to 1930 devote considerable space to the subject of invariants, the recent ones written from the axiomatic point of view disregard it completely. Because of this neglect the phrase "invariant theory" is apt to suggest a subject that was once of great interest but one that has little bearing on modern algebraic developments. For this reason it is an important and original accomplishment that Professor Weyl has made here in connecting the theory of invariants with the main stream of algebra and in indicating that the subject has a future as well as a distinguished past.

In his treatment the theory of invariants becomes a part of the theory of representations. The most natural way to begin the study of invariants of a particular group is therefore to determine its representations. A large part of the book is concerned with this problem as it applies to the "classical" groups $GL(n)$, the full linear group; $O(n)$, the orthogonal, and $S(n)$, the symplectic (complex or abelian) group.

In discussing the representations of an abstract group $\mathfrak{g}$ one finds it convenient to adjoin to the set of representing matrices their linear combinations. The resulting set is an algebra, the enveloping algebra of the original set, and defines a representation of a certain abstract algebra, the group algebra, that is completely determined by $\mathfrak{g}$ and the field of the coefficients. In this way the theory of algebra is applicable. The author has gone somewhat beyond his immediate needs in discussing this domain. Consequently his book may serve also as an excellent introduction to this theory.

The book begins with a brief review of the basic concepts of field, abstract group, vector space, linear transformation (matrix) and representation of a group. These are used to give the following definition of an invariant: Let $x, y, \cdots$ be variable vectors varying in different representation spaces of an abstract group $\mathfrak{g}$ and let $A(s), B(s), \cdots$ be the corresponding matrices. A polynomial $f(x, y, \cdots)$ in the coordinates of $x, y, \cdots$ is an invariant of $\mathfrak{g}$ if $f(A(s)x, B(s)y, \cdots) = f(x, y, \cdots)$ for all s in $\mathfrak{g}$. Relative invariants and covariants are defined along similar lines. The central problems to which this definition gives rise are: (1) Given a class of invariants, to determine, if possible, a basis, that is, a finite number $I_1, I_2, \cdots, I_r$ of these invariants such that every invariant in the class is expressible as a polynomial in the I's. (2) To determine a fundamental set of relations obtaining between the basic invariants. The solution of these prob-