To separate real and imaginary parts we may write

$$(f_1' + if_1)(f_2' + if_2) = f_1'f_2' - f_1f_2 + i\frac{d}{dx}(f_1f_2).$$

Differentiating this last expression p times and taking absolute magnitudes, we obtain

$$\left\{\frac{d^{p+1}}{dx^{p+1}}(f_1f_2)\right\}^2 + \left\{\frac{d^p}{dx^p}(f_1'f_2' - f_1f_2)\right\}^2 \leqq (2^p)^2,$$

which is more than we set out to prove. The functions $f_1(x) = \sin(x+\alpha_1)$ and $f_2(x) = \sin(x+\alpha_2)$ show that our constant is the "best possible."

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ON THE CARATHÉODORY CONDITION FOR UNILATERAL VARIATIONS[1]

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The two formulations and proofs of the Carathéodory condition in the calculus of variations given by Graves[2] do not necessarily apply to the case when the minimizing curve may have arcs in common with the boundary of the region of admissible variations.[3] The purpose of this note is to show how his first formulation and proof can be modified so as to be applicable to unilateral (one-sided) variations in the plane.

An admissible curve

$$E_0: \quad x^\alpha = x^\alpha(t), \qquad t_0 \leqq t \leqq T, \alpha = 1, 2, \cdots, n,$$

which minimizes the integral

$$J = \int_{t_0}^{T} F(x_1, \cdots, x_n, x_1', \cdots, x_n')dt \equiv \int_{t_0}^{T} F(x, x')dt$$

in the class of all admissible curves joining two fixed points x_0 and X in space of n dimensions $(n>1)$, must satisfy certain well known con-

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[1] Presented to the Society, December 29, 1939.

[2] *Discontinuous solutions in space problems of the calculus of variations*, American Journal of Mathematics, vol. 52 (1930), pp. 13–19.

[3] Cf. Mancill, *The minimum of a definite integral with respect to unilateral variations*, Contributions to the Calculus of Variations, 1933–37, University of Chicago, 1937, p. 121, condition C_{3C}.