A RELATION BETWEEN NON-ALTERNATING AND INTERIOR TRANSFORMATIONS[1]

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Recently in proving certain existence theorems for non-alternating and for interior transformations of a continuum onto a simple arc[2] it was observed that when a transformation of one of these types was set up, usually it satisfied in large measure conditions which brought it also under transformations of the other type. This suggests the existence of common ground shared by these sorts of transformations, and it is the object of this paper to exhibit the nature of such. Our principal conclusion is to the effect that, under certain auxiliary conditions, any interior transformation $f(M)=D$ of a compact continuum M onto a dendrite D can be factored into the form $f=f_2f_1$ where $f_1(M)=M'$ merely shrinks sets of type $f^{-1}(p)$ (p an end point of D) into single points and is topological elsewhere and $f_2(M')=D$ is non-alternating and interior. However, we first prove two theorems on the relation between the non-alternating property of a transformation of a continuum into a dendrite and the connectedness of the inverse sets for the end points of the dendrite.

THEOREM 1. *If M is a compact metric continuum and $f(M)=N$ is non-alternating, then for any end point*[3] *p of N, $f^{-1}(p)$ is connected.*

PROOF. Suppose $f^{-1}(p)$ is not connected. Then there exist points a and b of $f^{-1}(p)$, a closed set $F\subset M-f^{-1}(p)$ and a separation $M-F=M_a+M_b$ where $a\in M_a$, $b\in M_b$. Since $f(F)\cdot p=0$, $f(F)$ is compact, and p is an end point, there exists a neighborhood U of p in N with $\overline{U}\cdot f(F)=0$ and such that the boundary of U is a single point q. Since q separates p and $f(F)$ in N, it follows by a result of the author's[4] that there exists a separation $M-f^{-1}(q)=M_F+M_p$, where $F\subset M_F$, $f^{-1}(p)\subset M_p$. But this gives at once the separation $M-f^{-1}(q)=M_a\cdot M_p+(M_F+M_b\cdot M_p)$ where $M_a\cdot M_p\supset a$, $(M_F+M_b\cdot M_p)\supset b$, contrary to the hypothesis that f is non-alternating.

THEOREM 2. *If $f(M)=D$ is an interior transformation of a compact*

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[2] See my papers *The existence of certain transformations*, Duke Mathematical Journal, vol. 5 (1939), pp. 647–655; and *Non-alternating interior retracting transformations*, Annals of Mathematics, (2), vol. 40 (1939), pp. 914–921.

[3] That is, a point of Menger-Urysohn order 1 of N.

[4] See my paper in the American Journal of Mathematics, vol. 56 (1934), pp. 294–302, (1.4).