size of the eliminants but the size of their largest prime factor which is important, and secondly it is not essential to take the m's in order of magnitude. In answer, it should be pointed out that after one passes the limits of factor tables, it becomes impracticable to deal with the factors of the eliminant rather than the eliminant. Therefore, since the eliminant (in one case at least) appears to be an increasing function of m, one is compelled to work with monotone increasing m.

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SOME UNIFORMLY CONVEX SPACES

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1. **Introduction.** A Banach space is said to be uniformly convex if to every ϵ, $0<\epsilon<1$, there is a $\delta(\epsilon)$, $0<\delta(\epsilon)<2$, such that $\|x\|=\|y\|=1$ and $\|x-y\|\geqq\epsilon$ imply $\|x+y\|<2-\delta(\epsilon)$. J. A. Clarkson, who introduced the concept of uniform convexity [5], proved that the spaces L^p and l^p are uniformly convex if $p>1$, basing his proof on the following inequalities[2] among norms of elements of L^p or l^p:

$$\|x+y\|^p+\|x-y\|^p \leqq 2^{p-1}(\|x\|^p+\|y\|^p), \qquad p\geqq 2; \tag{1.1}$$

$$\|x+y\|^p+\|x-y\|^p \leqq 2(\|x\|^{p'}+\|y\|^{p'})^{p-1}, \qquad p\geqq 2; \tag{1.2}$$

$$\|x+y\|^{p'}+\|x-y\|^{p'} \leqq 2(\|x\|^p+\|y\|^p)^{p'-1}, \qquad 1<p\leqq 2. \tag{1.3}$$

The uniform convexity of L^p and l^p follows by inspection from either (1.1) or (1.2) if $p\geqq 2$, and from (1.3) if $1<p\leqq 2$. As Clarkson observed, (1.1) is a consequence of (1.2), since $\{(1/2)(a^r+b^r)\}^{1/r}$ is an increasing function of r for positive a and b [6, p. 26], so that the right side of (1.1) is not less than that of (1.2). However, (1.1) is interesting because it is considerably simpler to prove than (1.2) (see §3), so that the uniform convexity of L^p and l^p can be established more easily for $p\geqq 2$ than for $1<p<2$.

In this note I give a short proof of Clarkson's inequalities (and of a general set of inequalities, which includes them), using M. Riesz's convexity theorem for linear forms. This proof has the advantage that it can be generalized to show that the spaces $L^p\{L^q\}$, $L^p\{l^q\}$, $l^p\{L^q\}$, $l^p\{l^q\}$ are all uniformly convex[3] if $p>1, q>1$. Here $L^p\{E\}$ is

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[2] Here, as throughout this note, $p'=p/(p-1)$; similarly for other letters.

[3] These results suggest the possibility that $L^p\{E\}$ and $l^p\{E\}$ are uniformly convex whenever E is; but I can offer no evidence for or against this conjecture.