SOME PROBLEMS IN INTERPOLATION BY CHARACTERISTIC FUNCTIONS OF LINEAR DIFFERENTIAL SYSTEMS OF THE FOURTH ORDER

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In this paper we consider the convergence to $f(x)$, defined on $[0, 1]$, of

$$\Sigma_p[f(x)] = \alpha_{0p}u_0(x) + \alpha_{1p}u_1(x) + \cdots + \alpha_{pp}u_p(x),$$

where $u_n(x)$, $(n=0, 1, \cdots, p)$, are characteristic functions of certain self-adjoint linear differential systems of fourth order,

$$\alpha_{np} = \sum_{k=0}^{p}{}' f(x_k)u_n(x_k)\left\{\sum_{k=0}^{p}{}' u_n^2(x_k)\right\}^{-1}, \qquad n = 0, 1, \cdots, p,$$

and the symbol $\sum'$ is used in the sense $\sum_{k=0}^{\prime p} y_k = y_0/2 + \sum_{k=1}^{p} y_k$. Throughout the discussion, $x_k = 2k/(2p+1)$, $(k=0, 1, \cdots, p)$. The differential systems considered are

$$u^{(\mathrm{iv})} - \rho^4 u = 0,$$

with boundary conditions

I. $u'(0)=0,\ u'''(0)=0,\ u'(1)=0,\ u'''(1)+u(1)=0,$
II. $u'(0)=0,\ u'''(0)=0,\ u'(1)+u(1)=0,\ u'''(1)+u''(1)=0,$
III. $u(0)=0,\ u''(0)=0,\ u(1)=0,\ u''(1)+u'(1)=0,$
IV. $u'(0)=0,\ u'''(0)=0,\ u(1)=0,\ u''(1)+u'(1)=0,$
V. $u(0)=0,\ u'(0)=0,\ u(1)=0,\ u'(1)=0,$
VI. $u'(0)=0,\ u'''(0)=0,\ u(1)=0,\ u'(1)=0.$

The following theorems may be proved for these systems respectively.

I, II. *If $f(x)$ is continuous and of bounded variation in $[0, 1]$, then* $\lim_{p\to\infty}\Sigma_p[f(x)]=f(x)$ *uniformly in* $[0, 1]$.

III. *If $f(x)$ is continuous and of bounded variation in $[0, 1]$ and* $f(0)=f(1)=0$, *then* $\lim_{p\to\infty}\Sigma_p[f(x)]=f(x)$ *uniformly in* $[0, 1]$.

IV. *If $f(x)$ is continuous and of bounded variation in $[0, 1]$ and* $f(1)=0$, *then* $\lim_{p\to\infty}\Sigma_p[f(x)]=f(x)$ *uniformly in* $[0, 1-\eta]$.

V, VI. *If $f(x)$ satisfies a Lipschitz condition in $[0, 1]$ and* $f(0)=f(1)=0$, *then* $\lim_{p\to\infty}\Sigma_p[f(x)]=f(x)$ *uniformly in* $[\eta, 1-\eta]$.

Here and hereafter $\eta>0$ is arbitrarily small but fixed.

The method of proof for these theorems, as well as for those to fol-