RELATIONS AMONG THE FUNDAMENTAL SOLUTIONS OF THE GENERALIZED HYPERGEOMETRIC EQUATION WHEN $p=q+1$

II. LOGARITHMIC CASES*

F. C. SMITH

1. **Introduction.** In a previous paper [1], the author gave the relations among the non-logarithmic solutions of the equation

$$(1) \qquad \left\{ \prod_{t=1}^{q+1} (\theta + a_t) - \frac{1}{z} \prod_{t=1}^{q+1} (\theta + c_t - 1) \right\} y = 0$$

where $\theta = z(d/dz)$ and where the a_t and c_t are any constants, real or complex, the only restriction being that one of the c_t must be equal to unity. If no two of the a_t or c_t are equal or differ by an integer, then equation (1) has $q+1$ linearly independent solutions about the point $z=0$ which may be written

$$(2) \qquad Y_{0j} = z^{1-c_j} \prod_{t=1}^{q+1} \frac{\Gamma(1 + c_t - c_j)}{\Gamma(1 + a_t - c_j)} \sum_{n=0}^{\infty} \prod_{t=1}^{q+1} \frac{\Gamma(1 + a_t - c_j + n)}{\Gamma(1 + c_t - c_j + n)} z^n,$$
$$j = 1, 2, \cdots, q + 1; \ |z| < 1,$$

and $q+1$ linearly independent solutions about the point $z = \infty$ which may be written

$$(3) \qquad Y_{\infty j} = z^{-a_j} \prod_{t=1}^{q+1} \frac{\Gamma(1 - a_t + a_j)}{\Gamma(1 - c_t + a_j)} \sum_{n=0}^{\infty} \prod_{t=1}^{q+1} \frac{\Gamma(1 - c_t + a_j + n)}{\Gamma(1 - a_t + a_j + n)} \frac{1}{z^n},$$
$$j = 1, 2, \cdots, q + 1; \ |z| > 1.$$

If, however, we assume that

$$c_2 - c_1 = l_1; \ c_3 - c_2 = l_2; \ \cdots ; \ c_r - c_{r-1} = l_{r-1}$$

where each l_v is zero or a positive integer and assume at the same time that none of these r c_t is equal to or differs from any of the a_t by an integer, then the author has shown that the first r of the solutions (2) are replaced by the following forms [2]:

$$(4) \qquad Y_{0j} = \sum_{v=1}^{j} z^{1-c_v} \frac{(j-1)!}{(j-v)!} \left[\frac{\partial^{j-v}}{\partial w^{j-v}} z^w G_v(w, z) \right]_{w=0},$$
$$j = 1, 2, \cdots, r; \ |z| < 1,$$

where

* Presented to the Society, December 28, 1938.