ON THE BASIS THEOREM FOR INFINITE SYSTEMS OF DIFFERENTIAL POLYNOMIALS*

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Introduction. Let $\mathcal{J}$ be a differential field of characteristic zero.† We consider an infinite system Σ of differential polynomials in the letters $y_1, \cdots, y_n$, the coefficients of the differential polynomials being in $\mathcal{J}$.‡

A finite set Φ of forms in Σ is called a *basis* of Σ if, for every form G in Σ, there is a positive integer p, dependent on G, such that G^p is in the differential ideal of Φ. If a single p will serve for every G in Σ, then we shall call the basis *strong*.

It has been shown that every system has a basis.§ Raudenbush has shown further,‖ that there exist systems, not every basis of which is strong. It is now natural to ask whether or not every system of forms contains at least one strong basis.

We answer this question in the negative by showing that even a perfect differential ideal of forms may have no strong basis. The perfect differential ideal with which we work is the one generated by the form uv in the two unknowns u, v.

We employ several ideas used by Raudenbush in the second of his above mentioned papers.

1. **The assumption.** Consider a form¶ G every term of which is divisible by some u_iv_j.** Let Σ be the set of all such forms G. Then Σ is a differential ideal, and is perfect. For, if a form has a term free of, say, every u_i, then every power of the form will have such a term.

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† For the definition of differential field, and other terms, see H. W. Raudenbush, *Ideal theory and differential equations*, Transactions of this Society, vol. 36 (1934), pp. 361–368.

‡ Throughout the rest of this paper we shall use, as is customary, the term *form* for *differential polynomial.*

§ For differential fields of meromorphic functions this was essentially shown by J. F. Ritt in his book *Differential Equations from the Algebraic Standpoint*, American Mathematical Society Colloquium Publications, vol. 14, New York, 1932. See especially §§ 7, 77. Following the work of Ritt, Raudenbush treated the case of the general differential field of characteristic zero by purely algebraic methods. See Raudenbush, loc. cit.

‖ *On the analog for differential equations of the Hilbert-Netto theorem*, this Bulletin, vol. 42 (1936), pp. 371–373.

¶ For $\mathcal{J}$ we can use any differential field of characteristic zero.

** Subscripts denote derivatives.