The proof is too complicated to be sketched here. However it is worth saying that what Gentzen does is to describe a means of attaching an ordinal number (less than ϵ_0) to any proof of number theory. He then describes how, if one had a proof of a contradiction, one could find a second proof of a contradiction having a smaller ordinal number than the first proof.

J. BARKLEY ROSSER

Reports of a Mathematical Colloquium. Series 2, no. 1. Edited by Karl Menger. Notre Dame University Press, 1939. 64 pp.

This booklet of seven papers begins a continuation of the earlier series of reports issued from 1928 to 1936 by the Vienna Colloquium under the leadership of Professor Menger.

"Stability of Limited Competition and Cooperation," by G. C. Evans and Kenneth May, deals with two producers. Under simplifying assumptions, conditions are found on the coefficients of the demand and cost functions in order that an equilibrium point be possible, that this point be competitive or cooperative, and that the equilibrium be stable. Under strong hypotheses, similar methods are applied to labor, leading to the conclusion that a union able to control the labor supply for a given industry can make the introduction of machinery unprofitable to the entrepreneur.

"On Linear Sets in Metric Spaces," by Karl Menger and Arthur Milgram, contains four theorems, of which a special consequence is the known theorem that, in a complete and convex metric space, any two distinct points are joined by a subset congruent to a segment of the euclidean line.

The third paper is "Partially Ordered Sets, Separating Systems and Inductiveness," by A. N. Milgram. It is unfortunate that this interesting and substantial paper gives the impression of having been written hurriedly and printed without proofreading. The results given seem to be new and significant.

Essentially, certain portions of the Dedekind theory of the continuum are so adjusted as to be useful in studying partially ordered sets. Let A be a partially ordered set. A subset L is called a lower section of A if $a \varepsilon L$ and $b<a$ imply $b \varepsilon L$; if B is a subset of A, the set of those elements a of A with the property that $a \leqq b$ for every b in B is called the under section of B. In terms of these concepts, it is found possible to define well-ordered subsets and to associate with each element of A a unique subset which is well-ordered and has other useful properties; this association is produced once with and once without an application of transfinite induction. Separation—analogous to that effected in the continuum by the rational numbers—is defined intrinsically and extrinsically, the equivalence of the definitions is proved, and the powers of well-ordered subsets are compared with the powers of systems of separating sets. If A has a denumerable separating system, it is shown that A can be mapped on an interval of the continuum in such a way that order-relations are preserved. Several applications are given, chiefly to problems in topology.

"Postulates for the Ratio of Division," by B. J. Topel, deals with a set of elements and a real-valued function f of trios of these elements. f is subject to postulates which arise in a natural way from the properties of the ratio in which a point divides a segment in elementary geometry. It is shown that the set can be so metrized that f is the quotient of two distances. Limiting processes, orientation, and f-preserving transformations are studied, and other sets of postulates are considered.

Frederick P. Jenks proposes a set of postulates for Bolyai-Lobachevsky geometry based on the operations of joining and intersecting, and shows that these postulates are sufficient for the usual discussion of betweenness.