V_m IN S_n WITH PLANAR POINTS $(m \geqq 3)$

N. COBURN

1. **Introduction.** In this paper we shall classify the m-dimensional Riemannian manifolds (V_m) which are imbedded in an n-dimensional space of constant curvature (S_n) and whose normal curvature locus consists solely of planar points ($m \geqq 3$). Under the assumption that the second fundamental tensors have principal directions, we easily prove Segre's theorem:* *V_m in S_n with axial points are V_m in S_{m+1} or have second fundamental tensor of rank one.* Our proof is not as general as Segre's since the above additional assumption is required. However, our method can be generalized to classify the V_m in S_n with planar points. This classification is accomplished by use of the ranks of any of the two second fundamental tensors, which determine the normal curvature locus, and certain of the Ricci vectors. Our principal result is: *If the rank of any of these second fundamental tensors is greater than two, then V_m in S_n with planar points are* (1) *V_m consisting of ∞^1 V_{m-1} imbedded in ∞^1 S_{m+1}; or* (2) *V_m consisting of ∞^1 V_{m-1} imbedded in ∞^1 S_m; or* (3) *V_m lying in S_{m+2}.*

2. **Notation.** Let the unit tangent vector fields of m mutually orthogonal nonisotropic congruences of V_m in S_n be denoted by

$$(2.1)\qquad \underset{c}{i^\kappa} = \underset{c}{\epsilon}\,\overset{c}{i^\kappa}, \qquad \underset{c}{\epsilon} = \pm 1, \qquad \kappa, \lambda, \mu = 1, \cdots, n,$$
$$\underset{c}{i^\kappa}\,\overset{d}{i_\kappa} = \underset{c}{\overset{d}{\delta}}, \qquad a, b, c = 1, \cdots, m.$$

According to whether ϵ is $+1$ or -1, we say that i^κ is in the positive or negative quadric of directions, determined by the first fundamental tensor of S_n ($a_{\lambda\mu}$)

$$(2.2)\qquad a_{\lambda\mu}\underset{c}{i^\lambda}\underset{c}{i^\mu} = \underset{c}{\epsilon}.$$

The subscript in (2.1) refers to the congruence (orthogonal index), the contravariant index κ to the S_n coordinate system, the δ to the Kronecker symbol. For the $(n-m)$ mutually orthogonal unit vectors in the local E_{n-m} which is perpendicular to the local tangent E_m of the V_m at a point P, we write

$$(2.3)\qquad \underset{p}{i^\kappa} \qquad p, q, r = m+1, \cdots, n.$$

* Most of the references are to Schouten-Struik, *Einfuhrung in die Neueren Methoden der Differentialgeometrie*, vols. 1 and 2. Noordhoff, Groningen, Batavia. Hence we shall merely indicate volume and page number: vol. 2, pp. 96, 99.