

It remains an unsolved problem whether or not generalizations of the theorems of this paper can be established which would include the border and frontier operators or which would not require that the space be dense in itself.

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A NOTE ON A PAPER BY J. A. TODD*

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In a recent paper† entitled *A note on the linear fractional group*, Todd obtained an abstract definition for the group $LF(2, 2^n)$ in terms of $n+2$ generators. Apparently he gave no consideration to the question of the independence of the defining relations, for they can be considerably simplified. First, in view of the condition $RS_i = S_{i+1}R$ (which is the same as $R^i S_0 R^{-i} = S_i$), the three generators U, R and S_0 are sufficient to generate the entire group. If we give a definition in terms of these three generators alone, the relations

$$S_i^2 = 1, \quad i \neq 0, \quad RS_i = S_{i+1}R$$

may be discarded, and any S_i ($i \neq 0$) which appears in the remaining conditions may be replaced by its definition in terms of R and S_0 . Next, the $C_{n-1,2}$ conditions $S_i S_j = S_j S_i$ can be replaced by the $n-1$ conditions

$$S_0 S_i = S_i S_0, \quad i = 1, 2, \dots, n-1.$$

For suppose $j-i = \alpha$. Then, from $S_0 S_\alpha = S_\alpha S_0$, we get

$$R^i (S_0 S_\alpha) R^{-i} = R^i (S_\alpha S_0) R^{-i}, \quad S_i S_j = S_j S_i.$$

Writing $S_0 S_i = S_i S_0$ in terms of R and S_0 only

$$S_0 R^i S_0 R^{-i} = R^i S_0 R^{-i} S_0, \quad (S_0 R^i S_0 R^{-i})^2 = 1.$$

Thus, for the three generators U, R and S_0 , we require only $n+5$ conditions

$$R^{2^n-1} = U^3 = (UR)^2 = (US_0)^2 = S_0^2 = 1, \quad (S_0 R^i S_0 R^{-i})^2 = 1,$$

$$R^n S_0 R^{-n+1} = S_0^{\alpha_0} R S_0^{\alpha_1} R S_0^{\alpha_2} \dots R S_0^{\alpha_{n-1}} R^{-n+2}.$$

But even these three generators are not independent. For the relation $(UR)^2 = 1$ permits us to consider U and R as being equivalent to

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