POINT SET OPERATORS AND THEIR INTERRELATIONS*

E. C. STOPHER, JR.

INTRODUCTION

The fundamental operators of this paper will be defined in terms of a postulated derived set function. This procedure, first formally suggested by F. Riesz,† has been adopted by Chittenden‡ and others.

The notation used here, which has definite advantages over the classical notation, was first suggested by Chittenden. It first appeared in print in work by Sanders§ and later in a paper by the author.|| Capital letters A, B, and so on, will denote arbitrary sets of points in the space S, J $(=cdS)$ will denote the isolated points of the space, and 0 $(=cS)$ the null set. Operators will be denoted by small letters d, i, and so on. Thus, for example, iA represents the interior of the set A, and dkA the derived set of the kernel of A.

Such an operator which defines a set, either null or non-null, will be called a *product operator*. The number of single operators which make up a product operator will be called the *order* of the product operator. A product operator is said to be *reduced* if it is shown to be equal to another product operator of lower order or to one of the same order but which is expressed in terms of operators which precede in the list of definitions in Part I. Those for which no reduction has been found will be called *unreduced*.

In Part I is presented a table of all the second order product operators indicating the reductions. Many other reduction formulas involving higher order operators have been found, but these will be omitted in this paper.¶ In Part II the space will be assumed to be dense in itself. With the aid of this additional postulate the theorem can be proved that all product operators of a given family can be expressed in a certain canonical form.

* Presented to the Society, April 9, 1937.

† F. Riesz, *Stetigkeitsbegriff und abstrakte Mengenlehre*, Atti del 4 Congresso Internationale dei Matematici, Rome, 1910, vol. 2, p. 18.

‡ E. W. Chittenden, *On general topology and the relation of the properties of the class of all continuous functions to the properties of space*, Transactions of this Society, vol. 31 (1929), pp. 290–321.

§ S. T. Sanders, Jr., *Derived sets and their complements*, this Bulletin, vol. 42 (1936), pp. 577–584.

|| E. C. Stopher, Jr., *Cyclic relations in point set theory*, this Bulletin, vol. 43 (1937), pp. 686–694.

¶ Found with proofs in the doctor's dissertation by the author, *Interrelations of a Family of Operators on Point Sets and their Canonical Representation*, State University of Iowa, 1937.