$$C = \begin{pmatrix} L & M \\ N & P \end{pmatrix},$$

$L$, $M$, and so on, being in $\Sigma_n$, it follows that $M' = -M$, $N' = -N$, $P = L'$ and conversely any matrix of this form satisfies the condition. If we specialize the indeterminates so that $M = N = 0$, we obtain a matrix with irreducible minimum polynomial of degree $n$. It follows that $\psi(x)$ for the general matrix has degree not less than $n$ and hence $\psi(x) = \phi(x)$ and $f(x) = [\phi(x)]^2$.

THEOREM. *If $B$ is a matrix of $2n$ rows and columns with elements in a field of characteristic not equal to 2 such that $R^{-1}B'R = B$, where $R$ is any non-singular skew symmetric matrix, then the characteristic polynomial $f(x)$ of $B$ has the form $[\phi(x)]^2$, where the coefficients of $\phi(x)$ are polynomials in the elements of $B$ and $\phi(B) = 0$. If the elements of the general matrix $B$ are regarded as indeterminates, then $\phi(x)$ is irreducible.*

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## THE EULER-MACLAURIN SUMMATION FORMULA*

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In this paper an extension of the classical Euler-Maclaurin summation formula† is made to multiple sums. Bernoulli polynomials and numbers of higher order as defined by Nörlund‡ enter into the formula and Bernoulli numbers of negative order enter into the proof. Nörlund obtains§ a formula for $\phi(x+\omega)$ in terms of Bernoulli numbers of higher order, and this is called by him an extension of the Euler-Maclaurin formula. His formula permits the ready building up of a simple sum. This is not true, however, of a multiple sum. Steffensen|| calls attention to the fact that a multiple sum can be reduced by summation by parts to a simple sum and the Euler-Maclaurin formula for the simple sum used. However, the function to be summed is changed by his suggested transformation and he develops no general formula, nor does he suggest the use of Bernoulli numbers of higher order.

The formula developed in the present paper is equally as easy of

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† See, for example, D. Seliwanoff, *Lehrbuch der Differenzenrechnung*, p. 48, where it is called the Euler sum formula. It is sometimes also called the Maclaurin sum formula. See W. B. Ford, *Studies on Divergent Series and Summability*.

‡ N. E. Nörlund, *Differenzenrechnung*, p. 129.

§ Loc. cit., p. 160.

|| J. F. Steffensen, *Interpolation*, p. 136.