NEW POINT CONFIGURATIONS AND ALGEBRAIC CURVES CONNECTED WITH THEM*

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1. **Introduction.** In the memorial volume† for Professor Hayashi, I studied an involutorial Cremona transformation in a projective S_r which is obtained as follows: Let $C_i=(ax)_i\lambda_i^2+(bx)_i\lambda_i+(cx)_i=0$, $(i=1, 2, \cdots, r)$, be r hypercones in S_r. Every value of λ_i determines a hypertangent plane to the cone C_i. Thus the parameters $\lambda_1, \lambda_2, \cdots, \lambda_r$ for the hypercones $C_1, C_2, \cdots, C_r$, in the same order, determine r hyperplanes which intersect in a point (x) of S_r. From this point (x) there pass, one for each of the r hypercones, r more tangent hyperplanes whose parameters $\lambda_1', \lambda_2', \cdots, \lambda_r'$ are in the same order uniquely determined by the set $\lambda_1, \lambda_2, \cdots, \lambda_r$, and hence are rational functions

$$\rho\lambda_i' = \phi_i(\lambda_1, \lambda_2, \cdots, \lambda_r), \qquad i = 1, 2, \cdots, r,$$

of the parameters λ. Conversely, the set $\lambda_1', \lambda_2', \cdots, \lambda_r'$ determines λ_i uniquely: $\sigma\lambda_i=\phi_i(\lambda_1', \lambda_2', \cdots, \lambda_r')$. If therefore the λ's and λ''s are interpreted as coordinates of points of euclidean spaces $E_r(\lambda)$ and $E_r'(\lambda')$, there exists an involutorial Cremona transformation between the two r-dimensional spaces. The order and fundamental elements of this involution were determined in the corresponding projective spaces S_r and S_r' and applications given for S_2 and S_3. These belong to a remarkable class of involutions which have the property that when in S_r and S_r'

$$P\ (\lambda_1, \lambda_2, \lambda_3, \cdots, \lambda_{r+1}), \qquad P'\ (\lambda_1', \lambda_2', \lambda_3', \cdots, \lambda_{r+1}')$$

are corresponding points and any number of transpositions between coordinates in the same columns is performed, say

$$Q\ (\lambda_1, \lambda_2', \lambda_3', \cdots, \lambda_i', \cdots, \lambda_r, \cdots, \lambda_{r+1}'),$$
$$Q'\ (\lambda_1', \lambda_2, \lambda_3, \cdots, \lambda_i, \cdots, \lambda_r', \cdots, \lambda_{r+1}),$$

then Q, Q' is always a couple of corresponding points of the involution.

To this class also belong the well known quadratic and cubic involutions in S_2, $\rho x_i' = 1/x_i$, $(i=1, 2, 3)$, and in S_3, $\rho x_i' = 1/x_i$, $(i=1, 2, 3, 4)$,

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† The Tôhoku Mathematical Journal, vol. 37 (1933), pp. 100–109. See also Commentarii Mathematici Helvetici, vol. 4 (1932), pp. 65–73.