PROPERTIES OF GENERALIZED DEFINITIONS OF LIMIT*

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1. **Introduction.** The theory of summability has been the subject of several excellent expository addresses† presented to this Society. These addresses have dealt largely with properties of matrix transformations

$$A: \qquad\qquad y_s = \sum_{t=0}^{\infty} A_{s,t} x_t,$$

which associate with certain sequences $x_0, x_1, x_2, \cdots$ of complex numbers the sequences $y_0, y_1, \cdots$ determined by use of a given matrix $A_{s,t}$ of complex constants.

It is my object to discuss, and to compare with the matrix transformations A, the kernel transformations

$$K: \qquad\qquad y(s) = \int_0^{\infty} K(s, t) x(t) dt,$$

which associate with certain complex-valued functions $x(t)$ defined over $0 < t < \infty$ the functions $y(t)$ determined by a given kernel $K(s, t)$ belonging to a certain class of complex-valued functions which we specify in §3. Transformations of this form were first studied by Silverman.‡ More recent contributions§ have been made by Knopp, Hill, Raff, and Day.

The point of view of the present study of kernel transformations is quite different from that of earlier ones. The earlier studies have started with either the Riemann or Lebesgue integral and the class X

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† W. B. Ford, this Bulletin, vol. 25 (1918–1919), pp. 1–15; R. D. Carmichael, ibid., vol. 25 (1918–1919), pp. 97–131; C. N. Moore, ibid., vol. 25 (1918–1919), pp. 258–276; W. A. Hurwitz, ibid., vol. 28 (1922), pp. 17–36; and C. N. Moore, ibid., vol. 37 (1931), pp. 240–250.

‡ L. L. Silverman, *On the notion of summability for the limit of a function of a continuous variable*, Transactions of this Society, vol. 17 (1916), pp. 284–294.

§ K. Knopp, *Zur Theorie der Limitierungsverfahren*, Mathematische Zeitschrift, vol. 31 (1929–1930), pp. 97–127; pp. 276–305. J. D. Hill, *A theorem in the theory of summability*, this Bulletin, vol. 42 (1936), pp. 225–228. H. Raff, *Lineare Transformationen beschrankter integrierbarer Funktionen*, Mathematische Zeitschrift, vol. 41 (1936), pp. 605–629; *Über lineare Integraltransformationen*, Monatshefte für Mathematik und Physik, vol. 45 (1937), pp. 379–393. M. M. Day, *Regularity of function-to-function transformations*, this Bulletin, abstract 44-9-332.