SCHOUTEN AND STRUIK ON DIFFERENTIAL GEOMETRY

Einführung in die neuren Methoden der Differentialgeometrie. By J. A. Schouten and D. J. Struik. 2d completely revised edition. Vol. 1. *Algebra und Übertragungslehre.* By J. A. Schouten. 12+202 pp. Vol. 2. *Geometrie.* By D. J. Struik. 12+338 pp. Groningen, Noordhoff, 1935 and 1938.

As a revision and extension of the authors' previous treatises, this second edition of their joint work should be compared, not merely with the first edition, a short monograph published in 1924, but also with Struik's *Grundzüge der mehrdimensionalen Differentialgeometrie in direkter Darstellung,* published in 1922 and now out-of-print. The monograph dealt primarily with the algebraic and analytic aspects of the subject, whereas Struik's book was concerned essentially with Riemannian geometry.

Perhaps the most striking difference between the present work and those cited is in the methods employed. Schouten's system of direct analysis was used concurrently with the method of tensors in the original monograph and was employed almost exclusively in Struik's *Grundzüge.* In the book under review, this system has been completely discarded and tensor analysis given full sway.

In the notation for the components of geometrical objects Schouten's *Kern-Index-Methode* is consequentially applied throughout the treatise. The essence of this method consists in representing the effect on the components of a geometrical object due to a change of the system of reference, not by a different or primed central letter with indices of the original type, as is frequently the custom, but by the original central letter with indices of a different type. The change in the central letter with the preservation of the type of indices is reserved to represent the transformation of the object resulting from an actual transformation of the space. The method seems logically sound in principle and makes for definiteness and conciseness in practice. Readers to whom it is new will find that it is readily followed once they accustom themselves to scanning each formula for the full import of all the letters, central and appended, that are involved.

The outstanding advance in general methods since the appearance of the authors' first books has undoubtedly been the exploitation of nonholonomic systems of reference. These are introduced at the earliest opportunity in the first volume of the new work and are used throughout wherever fitting. In fact, the ordinary sign of equality is employed to indicate the validity of a relationship for all types of systems of reference. To indicate that an equation is true only for holonomic systems, an h is placed above the sign of equality, and a * serves a similar purpose in the case of an equation which holds only for the specific system of reference in use at the moment. Here, also, the ends of clarity and conciseness are well served.

Whereas the volumes of the early twenties treated only ordinary Riemannian geometry, the present work covers also the general linear connection and Hermitian connections, as well as the extension of Riemannian geometry to the general case in which the metric is not necessarily definite. In particular, in the latter connection, isotropic subspaces are given more than the customary passing mention.

The first volume begins with the algebraic foundation, including the fundamentals of affine geometry and of unitary geometry. The second part of this volume, in which the tools from analysis are developed, brings in at once nonholonomic systems of reference, develops the general linear connection with respect to them, discusses the D-symbolism of van der Waerden and Bortolotti in relation to the machinery for