

evaluated for $w=0$; in (30) we make special conventions of the same type as those made in connection with (13).

In connection with Theorem 4, it is of interest to note the unexpanded forms corresponding to (20), namely,

$$(31) \quad Y_{\infty j} = \sum_{v=1}^j z^{-av} \frac{(j-1)!}{(j-v)!} \left[\frac{\partial^{j-v}}{\partial w^{j-v}} z^{-w} F_v(w, z) \right]_{w=0},$$

$j = 1, 2, \dots, s.$

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ON THE FIRST CASE OF FERMAT'S LAST THEOREM*

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We prove the following theorem:

THEOREM. *If p is an odd prime, α, β , and γ are integers in the field of the p th roots of unity, $\alpha\beta\gamma$ is prime to p , and*

$$\alpha^p + \beta^p + \gamma^p = 0,$$

then $p \geq 8,332,403$.

As ordinary integers are integers in the field of the p th roots of unity, we infer the following:

COROLLARY. *The equation*

$$x^p + y^p + z^p = 0$$

has no solution in integers prime to p if p is an odd prime less than 8,332,403.

To abbreviate statements, we shall say that an odd prime p is improper if there are integers α, β , and γ in the field of the p th roots of unity such that $\alpha\beta\gamma$ is prime to p and

$$\alpha^p + \beta^p + \gamma^p = 0.$$

Then the theorem to be proved can be stated in the form:

THEOREM. *There are no improper odd primes less than 8,332,403.*

The proof is based on a theorem of Morishima† which, in our

* Presented to the Society, February 25, 1939.

† Taro Morishima, *Über die Fermatsche Vermutung*, Japanese Journal of Mathematics, vol. 11 (1935), pp. 241-252. Earlier results of a similar nature are due to Pollaczek, Frobenius, Vandiver, Mirimanoff, and Wieferich. Compare Dickson's *History of the Theory of Numbers*.