TOTALLY GEODESIC EINSTEIN SPACES*

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1. **Introduction.** An Einstein space† E_m is defined as a Riemann space V_m whose mean curvature a is a constant at each point; that is,‡

$$(1.1) \qquad R_{\alpha\beta} = - a g_{\alpha\beta}$$

where $R_{\alpha\beta}$ and $g_{\alpha\beta}$ are the Ricci and metric tensors of V_m, respectively. We suppose that the dimension m of E_m exceeds 3. For every surface is an E_2 and the only E_3's are the spaces of constant curvature. In both these cases, the discussion which parallels that given in this note is obvious and trivial. Since $m>3$, it is a well known consequence of (1.1) that a is a constant throughout the space. In this note, we discuss the properties of an E_m which admits families of totally geodesic subspaces which are also Einstein spaces. It is shown that this subject is closely related to the problems of finding (a) all Einstein spaces which may be imbedded as hypersurfaces of a space of constant curvature (b) Einstein spaces which are conformal to Einstein spaces. In a restricted sense,§ we also find the first fundamental form of E_m. It is assumed that the first fundamental forms of E_m and of its subspaces which are discussed below are nonsingular although they may be indefinite.

2. **Separable Einstein spaces.** It has been shown by Bompiani‖ that the necessary and sufficient condition that the subspaces x^p = const. and the orthogonal subspaces x^i = const. be totally geodesic in V_m is that

$$(2.1) \qquad g_{ij} = f_{ij}(x^k), \qquad g_{pq} = h_{pq}(x^r), \qquad g_{ip} = 0.$$

When the first fundamental form of V_m satisfies (2.1), it is called

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† We represent an m-dimensional Riemann space and Einstein space by V_m and E_m, respectively.

‡ Throughout this note, $\alpha, \beta, \gamma, \delta$; h, i, j, k; p, q, r; λ, μ, ν have the ranges $1, 2, \cdots, m$; $1, 2, \cdots, n$; $n+1, n+2, \cdots, m$; $1, 2, \cdots, n-1$, respectively. An index which appears twice in an expression is to be summed over the appropriate range. A free index of a tensor equation assumes each value of its range.

§ The first fundamental form of E_m is obtained in a preferred coordinate system and depends upon the unknown first fundamental form of an arbitrary Einstein space.

‖ E. Bompiani, *Spazi Riemanniani luoghi di varietà totalmente geodetiche*, Rendiconti del Circolo Matematico di Palermo, vol. 48 (1924), p. 124.