ON NON-BOUNDARY SETS*

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The purpose of this note is largely methodological; namely, to complete the triology of dense, boundary, and nondense sets by adding non-boundary sets.

We adhere to the nomenclature of Kuratowski's *Topologie*† except as noted. In particular, we suppose that S is a nonvacuous space satisfying his axioms of closure, and we write $F(X)=\overline{X}\cdot\overline{CX}$ and $X^0=C\overline{CX}$ where $CX=S-X$. If the set X has the property P, we write X^P or $(X)^P$, and in the contrary case X^{CP} or $(X)^{CP}$.

A set is *dense* if its closure is the space; *boundary* if its complement is dense; *nondense* if its closure is boundary; and finally, *non-boundary* if its complement is nondense. We designate the properties by D, B, ND, and NB, respectively.

THEOREM. *The following conditions are necessary and sufficient in order that a set be*

I. *Dense: The interior of its closure is the space; the boundary of its complement is the closure of its complement; its complement is a boundary set; its closure is a non-boundary set.*

II. *A boundary set: The closure of its interior is null; its boundary is its closure; its complement is dense; its interior is nondense.*

III. *Nondense: The interior of its closure is null; the boundary of its closure is its closure; its complement is a non-boundary set; its closure is a boundary set.*

IV. *A non-boundary set: The closure of its interior is the space; the boundary of the closure of its complement is the closure of its complement; its complement is nondense; its interior is dense.*

We summarize this in the following table of equivalences. The Roman numerals correspond to the statements above, and each statement in a row is equivalent to every other statement in that row.

The proofs of these statements are as follows: Column 2 is a formulation of the definitions. In column 3 statement I 3 follows from I 2 since $S^0=S$; II 2 is equivalent to $X^0=0$, which is clearly the same as II 3; III 3 is the complement of III 2; IV 3 is IV 2.

As to column 4, we have for I 4

$$(\overline{X} = S) \rightarrow (\overline{X}\cdot\overline{CX} = \overline{CX}) \rightarrow (F(CX) = \overline{CX}).$$

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† C. Kuratowski, *Topologie* I, Warsaw, 1933.