$$f(x) = \int_0^\infty x(t)dp(t), \qquad x \in S. \tag{4.2}$$

Now (4.1) is a linear functional on R, and consequently a linear functional on S. Hence (4.2) states that every distributive functional on S is linear; but this is impossible unless S is finite-dimensional,* which it is not. This contradiction establishes the theorem.

NORTON, MASSACHUSETTS

ON FUNDAMENTAL SYSTEMS OF SYMMETRIC FUNCTIONS†

H. T. ENGSTROM

A set S of n polynomials over a field K, symmetric in n variables, $x_1, x_2, \cdots, x_n$, is said to form a fundamental system if any rational function over K, symmetric in these variables, can be expressed rationally in terms of the polynomials of S. In this paper we show that any n algebraically independent symmetric polynomials over a field K of characteristic zero form a fundamental system if the product of their degrees is less than $2n!$.

The result follows from a theorem due to Perron:‡

THEOREM 1. *Between $n+1$ polynomials (not constant), $f_1, f_2, \cdots, f_{n+1}$, in n variables, of degrees $m_1, m_2, \cdots, m_{n+1}$, respectively, there is always an identity of the form*

$$\sum C_{\nu_1\nu_2\cdots\nu_{n+1}} f_1^{\nu_1} f_2^{\nu_2} \cdots f_{n+1}^{\nu_{n+1}} \equiv 0,$$

where in each term,

$$\sum_{i=1}^{n+1} m_i\nu_i \leqq \prod_{i=1}^{n+1} m_i.$$

---

* Let every distributive functional on S be linear, where S is a topological vector space with the property (Q). If S is infinite dimensional, let $\{x_n\}$, $(n=1, 2, \cdots)$, be an infinite set of linearly independent elements. Since $\lim_{k\to\infty} k^{-1}x_n = \Theta$, we can choose $y_n \in S$, $(n=1, 2, \cdots)$, linearly independent, with $y_n \to \Theta$. We set $f(y_n)=1$, $f(x)=0$ when x is not a finite linear combination of the y_n, $f(ax+by)=af(x)+bf(y)$ for any $x \in S$, $y \in S$; then f is a distributive functional on S, and hence is linear on S. Since $y_n \to \Theta$, $f(y_n) \to 0$ as $n \to \infty$; but this contradicts $f(y_n)=1$. Consequently S is finite dimensional.

† Presented to the Society, February 25, 1939, under the title *A note on fundamental systems of symmetric functions.*

‡ O. Perron, *Bemerkung zur Algebra*, Sitzungsberichte der Bayerischen Akademie, mathematisch-naturwissenschaftliche Abteilung, 1924, pp. 87–101.