ON GREEN'S FUNCTIONS IN THE THEORY OF HEAT CONDUCTION IN SPHERICAL COORDINATES†

ARNOLD N. LOWAN

In a previous paper,‡ the writer derived the expressions for the Green's functions in the theory of heat conduction for an infinite cylinder and for an infinite solid, bounded internally by a cylinder.

The object of the present paper is to derive the appropriate Green's functions for a sphere and for an infinite solid bounded internally by a sphere. In both cases, we shall take the boundary condition in the form

$$\frac{\partial u}{\partial r} + hu = 0, \qquad r = a.$$

The case of a sphere. In this case we start with the expression

$$u(r, \theta, \phi, t; r_0, \theta_0, \phi_0) = \frac{1}{2(\pi kt)^{3/2}} e^{-R^2/4kt}, \tag{1}$$

where

$$R^2 = r^2 + r_0^2 - 2r_0 \cos \gamma, \tag{2}$$

γ being the angle between the radii from the origin to the points (r, θ, ϕ) and (r_0, θ_0, ϕ_0). The expression (1) is the point source solution of the differential equation of heat conduction in spherical coordinates.

The expression (1) may be written in the form§

$$\begin{aligned} u(r, \theta, \phi, t; r_0, \theta_0; \phi_0) &= \frac{1}{4\pi(rr_0)^{1/2}} \sum_{n=0}^{\infty} (2n + 1)P_n(\cos \gamma) \\ &\quad \cdot \int_0^{\infty} \alpha e^{-k\alpha^2 t} J_{n+1/2}(\alpha r_0) J_{n+1/2}(\alpha r) d\alpha. \end{aligned} \tag{3}$$

The corresponding Laplace transform

$$L\{u(t)\} = \int_0^{\infty} e^{-pt} u(t) dt = u^*(p)$$

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‡ This Bulletin, vol. 44 (1938), pp. 125–133. This paper will be referred to as A.N.L.

§ See Carslaw, *Mathematical Theory of Heat Conduction*, article 93.