CREMONA TRANSFORMATIONS WITH AN INVARIANT RATIONAL SEXTIC

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It is well known that a Cremona transformation T with ten or fewer F-points will transform a rational sextic S into a rational sextic S' when the F-points of T are all located at the nodes of S. I have shown (cf. [1], p. 248, (9); [2], p. 255, (5)) that, even though the number of transformations T of the type indicated is infinite, the transforms S' are all included in $2^{13}\cdot 31\cdot 51$ classes, the members of any one class being all projectively equivalent and a member of one class being projectively distinct from a member of another class. The sextic S itself is in one of these classes, T being then the identity. If S' is in the same class as S, and if C is the collineation which carries S' into S, then TC is a Cremona transformation of the same type as T which transforms S into itself. There is thus an infinite group of Cremona transformations which carry S into itself. If t is a parameter on S, the effect of an element of such a group on the points of S is represented by

$$(1) \qquad t' = \frac{at+b}{ct+d}, \qquad ad - bc \neq 0.$$

It is an obvious question as to whether transformations T, other than the identical collineation, exist for which (1) reduces to $t'=t$; that is, whether S can be a locus of fixed points of a transformation T. I had expressed the *opinion* that such transformations T do not exist (cf. [1], end of §3). It was therefore most interesting to find in a recent article of G. Pompili (Pompili [1]) a purported construction of such a transformation. However the examination, made in the following, of this transformation shows that the construction is fallacious.

Let S be a generic rational sextic with nodes at $p_1, \cdots, p_7$ and at A, B, C. Let H be a generic member of the pencil (H) of elliptic sextics with nodes at $p_1, \cdots, p_7, B, C$, the pencil being determined by S and the square of the cubic $(p_1 \cdots p_7BC)^3$. On H let g_B, g_C denote the pairs of points at the nodes. Then on this elliptic curve the equivalence

$$(2) \qquad T\colon P' - P \equiv g_B - g_C$$

determines a birational correspondence which, extended over the various members of the pencil (H), yields a Cremona transformation T of the plane. If (H'), (H'') are similar pencils