(11) $$KM \equiv 0 \pmod{2^{n-1}\cdot 9}.$$

Conversely (11) implies (9). Since (9) holds for the modulus $2^{n-2}\cdot 9M$, it follows similarly that (11) holds for the modulus $2^{n-2}\cdot 9$ with $M=2^{n-4}M_1$. Hence (11) will be true for the given modulus if $M=2^{n-3}M_1$. This supplies a proof by induction that (8) is a universal form for every $n\geqq 4$.

If, in addition,* M is divisible by every prime p where $3<p\leqq n$, we satisfy the necessary condition given by Dickson† for the form (8) to represent at least one set of n primes. The proof of the sufficiency of this condition still remains a challenge to the ingenuity of number theorists.

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RINGS AS GROUPS WITH OPERATORS

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1. **Introduction.** A module M $(0, a, b, \cdots)$ is a commutative group, additively written. Every correspondence of M onto itself, or part of itself, such that $a\rightarrow a'$, $b\rightarrow b'$ implies $a+b\rightarrow a'+b'$ defines an *endomorphism* of M. An endomorphism may be regarded as an operator θ on M subject to the postulates (i) $\theta a=a'$ is uniquely defined as an element of M, (ii) $\theta(a+b)=\theta a+\theta b$, $(a, b \in M)$. In particular, there exist a null operator 0 $(0M=0)$ and a unit operator ϵ $(\epsilon a=a, a \in M)$. Designate by Ω_M the set of all such operators, $0, \epsilon, \alpha, \beta, \cdots$. It is well known that if operations of $\oplus$ and $\odot$ be defined in Ω_M by $(\theta+\eta)a=\theta a+\eta a$ and $(\theta\eta)a=\theta(\eta a)$, $(a \in M)$, Ω_M forms a ring with unit element ϵ (*endomorphism ring* of M).‡ The equation $\theta=\eta$ means $\theta a=\eta a$ (all $a \in M$). A ring $R(M)$ is called a ring over M in case M is the additive group of $R(M)$. Correspondence of a set P onto a set Q (many-one) is written $P\sim Q$; if specifically one-one, $P\cong Q$. Corresponding operations in P, Q preserved under the map are indicated in parentheses; for example, $P\sim Q$ $(+)$. If a set T has the property that TP is defined in P, TQ in Q, and if, under a correspondence $P\sim Q$, $p\rightarrow q$ implies $tp\rightarrow tq$ $(t \in T, p \in P, q \in Q)$, we write $P\sim Q$ (T) (T-operator correspondence). If R is a ring, the two-sided ideal N of elements z of R such that $zr=0$ (all $r \in R$), is called the left annulling ideal of R.

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* For example, replace $6M$ in (8) by $2^w n!M$, $(w\geqq n-3)$.

† Loc. cit., p. 156.

‡ van der Waerden, *Moderne Algebra*, vol. 1, 2d edition, p. 146.