

## THE DYNAMICS OF GEODESIC FLOWS\*

GUSTAV A. HEDLUND

1. **Introduction.** Geodesic systems, particularly those on two-dimensional manifolds, have been a rich source in the determination and display of the possible types of macroscopic behavior of the motions of dynamical systems. In connection with the question of the existence of periodic motions, Poincaré [1]† investigated the geodesics on convex surfaces. Hadamard [1] has constructed open surfaces of negative curvature and proved the existence of interesting classes of geodesics on these surfaces. By an ingenious use of symbolism to characterize these geodesics, Morse [1] proved the existence of nonperiodic recurrent geodesics of discontinuous type. Birkhoff [1] has constructed closed surfaces of nonpositive curvature and has shown that, among many other types, there exist transitive geodesics on these surfaces.

There is another group of mathematicians who have made numerous contributions in connection with geodesic systems on surfaces of constant negative curvature. As will be seen, these surfaces have a close relationship with Fuchsian groups, and in addition to their work having other connections with these groups, Artin [1], Myrberg [1, 2, 3], Nielsen [1, 2], Koebe [1], and Löbell [1, 2, 3, 4] have derived many properties of the geodesics.

With the recent developments in ergodic theory, interest has been centered on those properties of geodesic flows associated with transitivity in some form, as for example, regional transitivity, metric transitivity, and mixture. The conditions under which regional transitivity holds have been greatly extended by Morse [3]. Geodesic systems have furnished some of the few known examples of metrically transitive dynamical systems (cf. Hedlund [1, 2], E. Hopf [1]). As will be indicated, a number of new results concerning transitivity can be added, both in the case of constant curvature and in the case of variable curvature.

An enormous body of results has been attained, and an hour is entirely inadequate to permit a description of all. For this reason I propose to restrict the discussion to transitivity properties of geodesic flows. It has been conjectured (Birkhoff [3], p. 370) that these are the important properties in that they are *general* in some

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† Numbers in brackets refer to the bibliography at the end of the paper.