$$\iint_S \left\{ \left(\frac{\partial u}{\partial r} \right)_Q - c \right\} dS_Q = -4\pi c,$$

while from (5), for $G \equiv 1$ and $H = w$, we have

$$\iint_S \Delta_2 w(Q) dS_Q = 0.$$

THEOREM 3. *If $(\partial u/\partial r) = c \neq 0$ on the open set Ω of the sphere S, and if Σ is a domain containing S and its interior, it is in no case possible to extend u harmonically across Ω into the portion of Σ exterior to S.*

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ON THE CLASS OF METRICS DEFINING A METRISABLE SPACE*

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Suppose we are given a metrisable† space E. Let M be the class of all allowable metrics on E. Let M_b, M_c, M_B, and M_C be, respectively, the classes of metrics in which the space is bounded, complete, totally bounded, and totally complete. The purpose of this note is to obtain systematically all possible theorems which state the equivalence of some topological property of E (such as compactness, or separability) to the existence or non-existence of metrics having some of the above properties. An example is the well known theorem:

In order that E be compact it is necessary and sufficient that it be complete in every allowable metric.

The problem may also be stated as follows: Using the four definitions as principles of classification and noting the inclusions $M_b \supset M_B \supset M_b M_C$ and $M_c \supset M_C \supset M_c M_B$, we may represent M as the sum of seven disjoint sets: (1) $M - M_b - M_c$, (2) $M_b - M_B - M_b M_c$, (3) $M_c - M_C$

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† A topological space will be called *metrisable* if it is possible to define its continuity properties by means of a metric. Any metric which serves this purpose will be called *allowable*, and the space in conjunction with such a metric will be called a *metric space*. A metric space will be called *bounded* if there is a finite upper bound to the distance between any pair of its points. It will be called *complete* if every Cauchy sequence converges. It will be called *totally bounded* if it is, for every positive number e, the sum of a finite number of sets of diameter less than e. It will be called *totally complete* if every bounded set is compact. See C. Kuratowski, *Topologie* I, pp. 82, 87, 91, 196.