RELATIONS AMONG THE FUNDAMENTAL SOLUTIONS OF THE GENERALIZED HYPERGEOMETRIC EQUATION WHEN $p=q+1$. I. NON-LOGARITHMIC CASES*

F. C. SMITH

It is the purpose of this paper to develop the relations among the non-logarithmic solutions of the equation

$$\left\{\prod_{t=1}^{p}(\theta+a_t)-\frac{1}{z}\theta\prod_{t=1}^{q}(\theta+c_t-1)\right\}y=0,\quad p=q+1, \tag{1}$$

where $\theta=z(d/dz)$. The results of this paper generalize those of Mehlenbacher,† who studied the case in which $p=2$ and $q=1$, and extend those of Barnes‡ who obtained the asymptotic developments of the non-logarithmic solutions of (1) for those cases in which $p<q+1$.

The succeeding analysis will be simplified if we rewrite equation (1) in the equivalent form

$$\left\{\prod_{t=1}^{q+1}(\theta+a_t)-\frac{1}{z}\prod_{t=1}^{q+1}(\theta+c_t-1)\right\}y=0,\qquad c_{q+1}=1. \tag{2}$$

If no two of the a_t or c_t are equal or differ by an integer, then equation (2) has $q+1$ linearly independent solutions about the point $z=0$, which may be written

$$Y_{0j}=z^{1-c_j}\prod_{t=1}^{q+1}\frac{\Gamma(1+c_t-c_j)}{\Gamma(1+a_t-c_j)}\sum_{n=0}^{\infty}\prod_{t=1}^{q+1}\frac{\Gamma(1+a_t-c_j+n)}{\Gamma(1+c_t-c_j+n)}z^n, \tag{3}$$
$$j=1,2,3,\cdots,q+1;\ c_{q+1}=1;\ |z|<1;$$

and $q+1$ linearly independent solutions about the point $z=\infty$ which may be written

$$Y_{\infty j}=z^{-a_j}\prod_{t=1}^{q+1}\frac{\Gamma(1-a_t+a_j)}{\Gamma(1-c_t+a_j)}\sum_{n=0}^{\infty}\prod_{t=1}^{q+1}\frac{\Gamma(1-c_t+a_j+n)}{\Gamma(1-a_t+a_j+n)}\frac{1}{z^n}, \tag{4}$$
$$j=1,2,3,\cdots,q+1;\ c_{q+1}=1;\ |z|>1.$$

---

* Presented to the Society, December 30, 1937. The logarithmic cases will be considered in a later paper.

† See this Bulletin, Abstract 42-5-169.

‡ E. W. Barnes, *The asymptotic expansions of integral functions defined by generalized hypergeometric series*, Proceedings of the London Mathematical Society, (2), vol. 5 (1907), pp. 59–116.