A REMARK ON REPRESENTATIONS OF GROUPS*

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The purpose of this short note is to remark that we can state an analog of a famous theorem of Frobenius† on the induced characters of a finite group also for the representations of a general group.‡ This extension has not yet been explicitly stated, so far as I know, although it can be quite easily verified.

Let $\mathfrak{g}$ be a group, and let $\mathfrak{h}$ be a subgroup (*of a finite or infinite index*) of $\mathfrak{g}$.

DEFINITION. Let $F(x)$ and $f(\xi)$ be almost periodic (a. p.) functions (with complex numbers as values) on $\mathfrak{g}$ and $\mathfrak{h}$ respectively. Then we define the compositions of $F(x)$ and $f(\xi)$ by

$$f \times F(x) = M_{\xi\epsilon\mathfrak{h}}[f(\xi)F(\xi^{-1}x)],$$
$$F \times f(x) = M_{\xi\epsilon\mathfrak{h}}[F(x\xi^{-1})f(\xi)];$$

where $M_{\xi\epsilon\mathfrak{h}}$ means the construction of the mean with respect to a variable ξ in $\mathfrak{h}$. *Here* $f\times F(x)$ *and* $F\times f(x)$ *are a. p. functions on* $\mathfrak{g}$, *and they are linear with respect to both factors*, $f(\xi)$ *and* $F(x)$.

If $\mathfrak{h}_1$, $\mathfrak{h}_2$, $\mathfrak{h}_3$ *are three subgroups of* $\mathfrak{g}$ *such that* $\mathfrak{h}_i \subseteqq \mathfrak{h}_k$ *or* $\mathfrak{h}_i \supseteqq \mathfrak{h}_k$ *for every* i, $k=1, 2, 3$, *then*

$$(f_1 \times f_2) \times f_3 = f_1 \times (f_2 \times f_3)$$

for a. p. functions $f_1(\xi_1)$, $f_2(\xi_2)$, $f_3(\xi_3)$ *on* $\mathfrak{h}_1$, $\mathfrak{h}_2$, $\mathfrak{h}_3$ *respectively.*

(Both sides of the equality are a. p. functions on the greatest among the $\mathfrak{h}_i$.) This product we denote by $f_1\times f_2\times f_3$.

All these statements we can prove by a procedure similar to that

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† G. Frobenius, *Ueber Relationen zwischen den Charakteren einer Gruppe und denen ihrer Untergruppen*, Berlin Sitzungsberichte, 1898; H. Weyl, *Gruppentheorie und Quantenmechanik*; J. Levitzki, *Ueber vollständig reduzible Ringe und Unterringe*, Mathematische Zeitschrift, vol. 33 (1931).

‡ J. von Neumann, *Almost periodic functions in a group*, Transactions of this Society, vol. 36 (1934). Cf. also S. Bochner and J. von Neumann, *Almost periodic functions in groups*, II, ibid., vol. 37 (1935); W. Maak, *Eine neue Definition der fastperiodischen Funktionen*, Abhandlungen aus dem Mathematischen Seminar, Hamburg, vol. 11 (1936); B. L. van der Waerden, *Gruppen von linearen Transformationen*, Ergebnisse der Mathematik, vol. 4 (1935).

By a representation of a group we understand always a bounded one in the field of complex numbers.