

interest given by Wicksell.\* It was obtained for the simplified system already mentioned, by Lange.† The remarks in this section are in fact a generalization or justification of this latter theory.

The index relations are particularly interesting in discussing changes from one system to another consequent on the introduction or change of interest rate  $\lambda$ . Thus, with the index of primary factors given, that is,  $Q_2$  given, the introduction of a small interest rate  $\lambda$  induces no modification of  $Q_3$  as far as differentials of the first order. In fact,

$$\delta Q_3 = 0,$$

and

$$\delta^2 Q_3 = \lambda \frac{\partial Q_3}{\partial Q_1^3} \delta Q_1.$$

Equations such as these are important for economic theory.

UNIVERSITY OF CALIFORNIA

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### NOTE ON ALMOST-UNIVERSAL FORMS‡

P. R. HALMOS

Ramanujan§ and Dickson|| proved that there are 54 universal forms  $ax^2+by^2+cz^2+dt^2$  with positive integral coefficients  $a, b, c, d$ . It is the purpose of this note to investigate *almost-universal forms*, that is, to exhibit sets of positive integral coefficients  $a, b, c, d$  such that  $ax^2+by^2+cz^2+dt^2$  represents every positive integer with exactly one exception.

Ramanujan§ showed that a necessary and sufficient condition that a form  $ax^2+by^2+cz^2+dt^2$  be universal is that it represent the first fifteen positive integers. Consequently the integer which an almost-universal form fails to represent cannot be greater than 15. Using Ramanujan's method of bounding the coefficients we can exhibit, merely by requiring that a form fail to represent exactly one of the

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\* Wicksell, *Lectures on Political Economy*, London, 1935 (translation), vol. 1, p. 156.

† Lange, loc. cit.

‡ Presented to the Society, December 28, 1937.

§ Proceedings of the Cambridge Philosophical Society, vol. 19 (1917), pp. 11-21; *Collected Papers*, Cambridge, 1927, pp. 169-178.

|| This Bulletin, vol. 33 (1927), pp. 63-70.