CONCERNING SETS OF POLYNOMIALS ORTHOGONAL SIMULTANEOUSLY ON SEVERAL CIRCLES*

G. M. MERRIMAN

1. **Introduction.** Starting with the well-known fact that the set of polynomials $\{z^n\}$ is an orthogonal set on every circle $|z|=R$, there has recently been some consideration of the general problem suggested, that of the existence of sets of polynomials in the complex variable which are orthogonal, with respect to suitable norm functions, simultaneously on more than one curve. Terming a set of polynomials $\{p_n(z)\}$ *canonical* on a rectifiable Jordan curve C with respect to the positive continuous norm function $n(z)$ provided it is found by orthogonalizing on C the set $\{z^n\}$ with respect to $n(z)$, and provided the coefficient of z^n in $p_n(z)$ is chosen positive, we list the previous results† which we shall find pertinent to our purpose:

(1) Walsh‡ and Szegö§ have shown, independently and by different methods, that if the same set of polynomials is canonical on two distinct curves then one of the curves is a "level curve" (Kreisbild)‖ in the conformal mapping of the region outside the other curve onto the exterior of a circle, the points at infinity corresponding to each other.

(2) Szegö¶ has exhibited *all* canonical sets of polynomials in the complex variable, each set canonical on *all* level curves of a given family; there are only five essentially different types of such sets.

While this last result is definitive in connection with sets of polynomials canonical simultaneously on a whole family of level curves, the general problem of the existence of sets of polynomials canonical simultaneously on only a *finite* number of curves has not yet been discussed; in the references cited above, Walsh (p. 136) and Szegö (p. 196) both suggest its study. It is the purpose of the present paper

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† A complete list of results on this problem will be found in §1 of Walsh and Merriman, *Note on the simultaneous orthogonality of harmonic polynomials on several curves*, Duke Mathematical Journal, vol. 3 (1937), pp. 279–288.

‡ J. L. Walsh, *Interpolation and Approximation by Rational Functions in the Complex Domain*, American Mathematical Society Colloquium Publications, vol. 20, p. 134, Theorem 11.

§ G. Szegö, *A problem concerning orthogonal polynomials*, Transactions of this Society, vol. 37 (1935), pp. 196–206, Theorem I.

‖ Cf. Walsh, loc. cit., §4.1.

¶ Loc. cit., pp. 197–198.