

of Rademacher and to the "probabilistic" interpretation of the latter. Chapter 5 (pp. 149-194) is devoted to the theory of convergence (almost everywhere, unconditional,  $\dots$ ), divergence, and summability of orthogonal series. The climax of this chapter is reached in an elegant proof of the fundamental theorem of Rademacher-Menchoff. A systematic use of "Lebesgue's functions" associated with orthogonal expansions deserves a special mention. Chapter 6 (pp. 195-242) deals with orthogonal expansions in various spaces ( $L^p$ ,  $C$ ,  $M$ ) different from  $L^2$ . Among various topics treated here we mention the relationships between the closure and completeness of an orthogonal system; the theorems of Young-Hausdorff and of Paley; the theory of "multipliers" transforming orthogonal expansions of functions of various classes into each other; and a discussion of various singularities which occur in orthogonal expansions. Chapter 7 (pp. 243-260) reveals various remarkable properties of "lacunary" series. Chapter 8 (pp. 261-298), the last chapter, is of somewhat mixed character, being devoted partly to biorthogonal expansions, and partly to polynomials orthogonal relative to a given weight-function.

The exposition, which is in general clear and concise, in some places shows a tendency to be either somewhat vague, or so condensed that it will be difficult to follow for a reader who is not well versed in the field. The number of misprints (in addition to those mentioned in a list of 16 Errata) and of slips of pen or thought is not entirely negligible. Thus on page 6 the reader is told that every point set can be decomposed into a sum of a perfect set and of an at most denumerable set; on page 19 the norm of the functional  $\int_a^b dg$  is stated to be  $\int_a^b |dg|$ ; the condition [874] on page 280, which is sufficient for the completeness of the system of orthogonal polynomials relative to the weight function  $w(t)$ , is obviously not satisfied in the case of Laguerre polynomials, contrary to the assertion preceding this condition. In order to avoid footnotes the authors are using a new scheme of cross references, which, according to the reviewer's experience, does not represent an improvement over the customary system.

J. D. TAMARKIN

*Introduction à la Théorie des Fonctions de Variables Réelles.* Parts I and II. By Arnaud Denjoy. (Actualités Scientifiques et Industrielles, nos. 451 (55 pp.) and 452 (57 pp.).) Paris, Hermann, 1937.

These are two of the brochures in the section on *Sets and Functions*, under the editorship of Denjoy; who here writes Parts I and II on the introduction to real function theory. Having himself gained important results (on derived numbers, totalization, and so on), Denjoy has paused to scan the field of sets and real variables. From the brevity of each brochure it is clear that his treatment of the various topics is necessarily skeletal (it omits all proofs), but, we believe, is interesting and successful.

Early in Part I he advances reasons of a physical nature for studying non-analytic, even discontinuous, functions. Then comes a foremention of names and topics to be considered: Cauchy, Abel, Riemann (on convergence and integration), Cantor (sets, transfinite numbers), Baire (classification of functions), Borel, Lebesgue (measure and integration),  $\dots$ ; and a mention of general analysis. Chapter 2 deals with the geometry of Cartesian point sets, that is, familiar point set theory (including measure). The author here makes distinction between descriptive (topological) ideas and metric ideas, which distinction is also carried over to Chapter 3 on the analysis of functions. Examples of descriptive notions are continuity, convergence, the Baire classification; of metric notions, derived numbers, differentials. Functions of