theorem quoted gives also the simplest condition, which is that the functions u and v, where $f=u+iv$, possess a Stolz differential and satisfy the Cauchy-Riemann differential equations. If the Stolz differential is assumed, then it is also sufficient either (a) that the difference quotient $\Delta f/\Delta z$ have the same limit for any two distinct directions, or (b) that arg $\Delta f/\Delta z$ have the same limit for three distinct directions, or (c) that $|\Delta f/\Delta z|$ have the same limit in three directions, but in the latter case $\bar{f}$ may be monogenic instead of f. The theorems giving sufficient conditions for the holomorphism of a function in a region depend upon sufficient conditions for the expression of the integral $\int f(z)dz$ around a rectangle with sides parallel to the axes in the form of the double integral

$$-\iint\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)dx\,dy+i\iint\left(\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}\right)dx\,dy$$

and the fact that if $\overline{\lim}\,|\Delta f/\Delta z|$ is finite on a measurable set, then the Stolz differential of f exists except on a set of measure zero, various theorems being obtained by giving sufficient conditions for the application of these results and the use of Morera's theorem.

In the main the monograph is a brief presentation of the author's investigations in these questions, which may be justifiable, but involves the possibility of overlooking simplifications in presentation. For instance, the three supplementary sufficient conditions for the monogeneity of a function at a point are geometrically intuitive if use is made of the Kasner circle.* However, the monograph is informative and suggestive; especially might one call attention to the remark in the introduction that, while many theorems have a form which involves only the complex variable situation, it has so far been necessary to use in their proof deep-seated methods of the modern theory of real functions, and that it would be interesting and desirable to derive these same theorems without departing from the setting in which they are stated.

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Théorie de l'Addition des Variables Aléatoires. By P. Lévy. (Monographies des Probabilités, publiés sous la direction de E. Borel, no. 1.) Paris, Gauthier-Villars, 1937. 17+328 pp.

Professor Lévy's book is the first of a series of monographs on probability and its applications. This first monograph covers the field of Lévy's special interest, to which he has made valuable and often definitive contributions: certain questions connected with asymptotic problems in probability theory. The first four chapters include a general introduction to probability, designed to make the book complete in itself. Chapter V contains theorems related to the Gaussian law—Cramer's recently proved theorem and various extensions of the Liapounoff theorem. In Chapter VI, Lévy discusses series whose terms are mutually independent chance variables. He includes an important theorem on dispersion, recently derived by himself and Doeblin. Chapter VII considers a chance variable x_t (depending on the parameter t) whose increments in non-overlapping t-intervals are independent. The treatment is somewhat confusing in that measurability considerations are omitted and it is not clear whether x_t considered as a function of t has been proved to have (almost certainly), at worst, jumps at its points of discontinuity. Lévy gives a considerably simplified version of

* This Bulletin, vol. 34 (1928), p. 561.