$$\frac{\partial P_i}{\partial \sigma} = A_i. \tag{14}$$

This system, together with the initial conditions, is satisfied by $P_i = 0$, $(i = 1, \cdots, k)$. Hence, on account of the uniqueness of the solution of (14) with given initial values, we conclude that $P_i \equiv 0$, and the proof is complete.

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ON THE EXISTENCE OF LINEAR FUNCTIONALS DEFINED OVER LINEAR SPACES*

BY R. P. AGNEW

1. *Introduction*. A function $q(x)$ with domain in a linear space E and range in the set R of real numbers is called a *functional*, and $q(x)$ is called *linear*, if

$$q(ax + by) = aq(x) + bq(y), \qquad x, y \varepsilon E;\ a, b \varepsilon R. \tag{1}$$

We call a functional $r(x)$ an *r-function* (over E) if there exists a linear functional $f(x)$ with

$$f(x) \leqq r(x), \qquad x \varepsilon E. \tag{2}$$

Using a notation of Banach† we call a functional $p(x)$ a *p-function* if

$$p(tx) = tp(x), \qquad t \geqq 0,\ x \varepsilon E, \tag{3}$$

$$p(x + y) \leqq p(x) + p(y), \qquad x, y \varepsilon E. \tag{4}$$

A fundamental theorem of Banach (loc. cit., p. 29) can be stated as follows:

THEOREM (Banach). *Each p-function is an r-function.*

In some problems‡ involving existence of linear functionals $f_1(x)$ having prescribed properties, there appears a functional $q(x)$ with the following significance: There exists a linear functional f_1 having the requisite properties if and only if there exists

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* Presented to the Society, September 8, 1937.

† S. Banach, *Théorie des Opérations Linéaires*, Warsaw, 1932, p. 28.

‡ The author intends to discuss these problems at some future time.