REMARKS ON THE INITIAL VALUE PROBLEM OF THE GENERAL PARTIAL DIFFERENTIAL EQUATION OF THE FIRST ORDER

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1. *Introduction.* This paper presents a new approach to the classical theory of partial differential equations of the first order by establishing the equivalence of the initial value problem to another problem associated with a system of quasi-linear partial differential equations of the first order. All the equations of this quasi-linear system will have the same "principal part"; this means that the derivatives have the same coefficients in all the equations. Incidentally, systems of differential equations with the same principal part are assuming an important rôle in the theory of partial differential equations.*

In the case of quasi-linear systems the theory of integration can, as we shall see, be reduced to that of the integration of a system of ordinary differential equations in the same way as with a single linear partial differential equation. We shall then show that the initial value problem of the general partial differential equation of the first order is equivalent to a certain initial value problem of our quasi-linear system with identical principal parts. In this way we can develop the theory of characteristics and the solution of the initial value problem for a general partial differential equation of the first order.

2. *Systems of Quasi-Linear Partial Differential Equations of the First Order with Identical Principal Parts.* Let

$$\sum_{i=1}^{k} a_i \frac{\partial u_\mu}{\partial x_i} = b_\mu, \qquad (\mu = 1, \cdots, m), \tag{1}$$

be a system of quasi-linear partial differential equations in which the coefficients a_i are the same in all equations. This is a system of quasi-linear equations with identical principal parts for m functions $u_1, \cdots, u_m$ of k variables $x_1, \cdots, x_k$. In these equations a_i and b_μ are functions of $x_1, \cdots, x_k, u_1, \cdots, u_m$.

* For this topic see Courant-Hilbert, *Methoden der Mathematischen Physik*, vol. II, Chapter 2, appendix and Chapter 5 (in press).