SOME SYMBOLIC IDENTITIES*

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Differential equations were first solved by symbolic methods in England and on the continent in the first half of the last century. The differentiation symbol was treated as a symbol of quantity with restrictions. Then followed symbolic treatment of invariants and covariants, Cayley's hyperdeterminant, and Aronhold's symbolic notations. These were followed by Blissard's umbral notation in the theory of numbers.

This paper is devoted to showing that these are all reducible to symbolic differentiation.

If we represent differentiation by the symbol D and separate the symbols of operation from the symbols of quantity, then any analytic identity, such as $\Phi_1(y)=\Phi_2(y)$, will give an operational identity, $\Phi_1(D)=\Phi_2(D)$.

If this operational identity is applied to a second identity

$$F_1(x) = F_2(x)$$

the result will be a new identity. Most identities obtained in this way are easily obtained otherwise. The following are some examples.

Invariants and covariants. If $D_1=\partial/\partial x_1$ and $D_2=\partial/\partial x_2$, then

$$D_2^r D_1^{n-r}(\alpha_1 x_1 + \alpha_2 x_2)^n = n!\alpha_1^{n-r}\alpha_2^r,$$

where α_x^n is a special form of degree n, while the operation on the general form gives

$$D_2^r D_1^{n-r} f(x_1, x_2) = D_2^r D_1^{n-r}(a_0 x_1^n + n a_1 x_1^{n-1} x_2 + \cdots) = n!a_r.$$

We now transform our coordinates:

$$x_1 = \xi_1 X_1 + \eta_1 X_2, \qquad x_2 = \xi_2 X_1 + \eta_2 X_2,$$

or

$$X_1 = \frac{1}{\Delta}(\eta_2 x_1 - \eta_1 x_2), \qquad X_2 = \frac{1}{\Delta}(-\xi_2 x_1 + \xi_1 x_2),$$

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