we have

$$\lim_{k\to\infty} \delta_i^{(k)} = \prod_j^{(i)} d_{i,j} \prod_j D_{i,j}.$$

Now

$$\left| \delta_i^{(k)} - \delta \right| = \delta \left| 1 - \frac{\prod_j^{(i)} D_{i,j}^{(k)}}{\prod_j^{(i)} D_{i,j}^{(k-1)}} \right|.$$

But the quantity on the right approaches zero, so that $\delta_i^{(k)} \to \delta$ as $k \to \infty$. We thus have (1), and the theorem is proved.

INSTITUTE FOR ADVANCED STUDY

AN INVOLUTORIAL LINE TRANSFORMATION IN S_4

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1. *Introduction.* It is a well known fact that all planes which meet four general lines of S_4* are met by a fifth line. The remarkable configuration determined by five such "associated lines" is discussed in a number of places in the literature.† In the present paper an involutorial line transformation suggested by the figure of five associated lines is discussed, both as a line involution in S_4, and as a point involution on a certain V_6^5 in S_9. In §§2–6 the involution is treated at some length by purely synthetic methods. The final section (§7) contains a brief analytic treatment, including the equations of the involution, and the equations of the invariant and singular elements. The involu-

* We shall use the conventional symbol S_m to indicate a linear space of dimension m. A variety of order r and of dimension m we shall designate by the symbol V_m^r.

† Welchman, W. G., *Plane congruences of the second order in space of four dimensions and fifth incidence theorems*, Proceedings of the Cambridge Philosophical Society, vol. 28 (1931–1932), pp. 275–284.

Baker, H. F., *On a proof of the theorem of a double six of lines by projection from four dimensions*, Proceedings of the Cambridge Philosophical Society, vol. 20 (1920–1921), pp. 133–144.

Baker, H. F., *Principles of Geometry*, Cambridge University Press, 1925, vol. IV, Chapter V,